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ABSTRACT

A quadrature generation technique is formulated on the unit sphere for discrete ordinate transport programs. A computer code is developed for automatic generation of weights and directions for arbitrary order n. Critical dimension predictions using the generated directed direction sets are compared with analytical transport solutions, P_{n-1}, $DP_{(n-2)/2}$, and EQ_n approximations in one-dimensional, one-group slabs, cylinders, and spheres. The k_{eff} of the six-energy group Lady Godiva spherical benchmark problem is critiqued using various set and solutions methods.

I. INTRODUCTION

Numerical solutions of the Discrete Ordinates S_n method have been used extensively for neutral and charged particle transport calculations. The basic S_n technique discretizes the particle direction variables of the linearized Boltzmann transport equation. The numerical approximation to the multigroup transport equation is made conservative by integrating over the phase volume with constant cross-section properties assumed within each mesh cell. The resultant coupled equations are approximated further by assuming a finite difference (FD) linear relationship between the cell centered and cell boundary values in the phase space. Solutions typically require considerable computational resources. The computational accuracy of the S_n method is limited by the assumption of constant material properties in each phase space cell, by the number of mean-free paths per cell, by the specific model of nearest neighbor coupling, and by the particular choice of discretized directions used in the calculations. In this paper, we focus on the development of different choices of discretized directions and upon the critical radii and k_{eff} eigenvalue predictions resulting therefrom.

The development of S_n quadrature sets has been investigated extensively.[1–8] These efforts were directed toward satisfying certain specific symmetry conditions, predicting moment conditions, evaluating extrapolated end-point predictions, direct benchmark computation, and developing generalized direction sets that could be extended to multi-dimensional geometries.

We develop an area method[2,3] on the unit sphere for the direct construction of positive-point weights and directions for an arbitrary number of discrete directions, n. The method has been automated into a CDC-7600 FORTRAN program, which can be used to efficiently produce the required direction sets for the standard one- and two-dimensional neutron transport codes. The rotation-reflection symmetry requirements first introduced into the S_n quadrature sets[2,3] and available in transport codes,[9–13] are derived and reviewed in Sec. II.

In Sec. III we explicitly construct a set of area method figures satisfying rotation-reflection invariance symmetry. In Sec. IV we review the counting procedure for the number of figures of each type. In Sec. V we exhibit examples of these constructed direction sets and the moment conditions that are satisfied.

Finally, in Sec. VI we compare the predicted results of these direction sets on two simple classes of problems using several different numerical methods available in a modified version of a single discrete ordinates S_n code.[11] First, we investigate critical thickness predictions and convergence as a function of n, for the one-group homogeneous slab, cylinder, and spherical geometries.[3] Second, using the various direction sets, we calculate and compare k_{eff} for the Lady Godiva

Benchmark problem,[14] based on Lady Godiva[15] using the Hansen-Roach six-group cross sections.[16] Approximately 2084 criticality calculations are reported and compared, 1824 on one-group slabs, cylinders, and spheres, and 260 benchmark comparisons are made on Lady Godiva.

II. ROTATION-REFLECTION SYMMETRY

A proper set of discrete directions, suitable for usage in the multidimensional numerical transport codes, may be constructed by placing a triangular set of points on a spherical octant. These points are located symmetrically relative to the three vertices with respect to $2\pi/3$ rotations about the octant midpoint and inversion about the spherical arcs connecting the midpoints and vertices. With the development of these discrete directions, one should not expect the full-rotational symmetry properties of the spherical harmonic expansion of the angular flux to be obtained.

The points are arranged in n/2 levels. As illustrated in Fig. 1, we define the i^{th} level from the side opposite any vertex to contain $(n/2) - i + 1$ points. These total $n(n + 2)/8$ points define the n^{th} order S_n discrete direction approximation on the octant. Specific applications may use from n (one-dimensional slab and sphere) to $n(n + 2)$ (full-sphere) points.

The axes on the unit sphere ($R^2 = 1$) in Fig. 1 are denoted OA, OB, and OC. The levels are defined as latitude circles with respect to a vertex.

The position, Q_s, of any discrete neutron direction is given in terms of the direction cosines, μ, of its level position from the three sides (BC, AB, and AC) by

$$Q_s = (\mu_i, \mu_j, \mu_k) \quad , \tag{1}$$

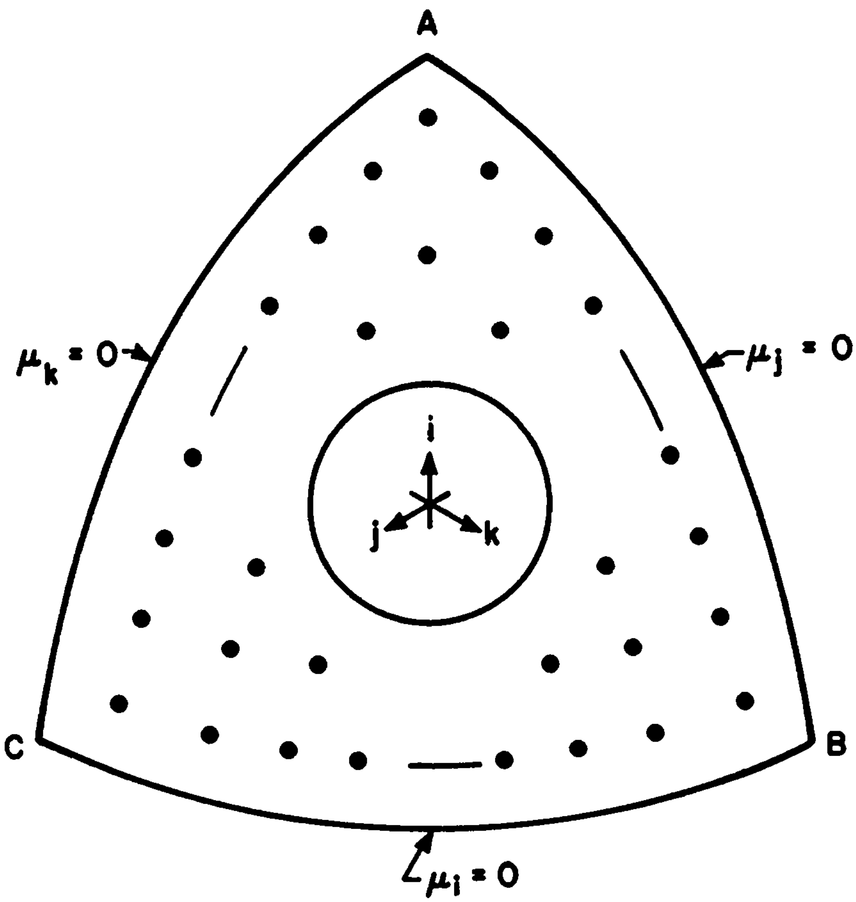

Figure 1. Arrangement of quadrature points on the octant.

where the indices satisfy the sum rule, $i + j + k = (n + 4)/2$, and the direction cosines μ_i, μ_j, and μ_k are the Q_s projections on the axes OA, OB, and OC, respectively. The coordinates of a point are denoted by the triad, $(j^i k)$ and are measured relative to the side opposite the named vertex. For example, in Fig. 2, the point $(1^1 4)$ closest to the vertex B has the representation $Q_s = (\mu_1, \mu_4, \mu_1)$.

Applying the law of direction cosines to Eq. (1) yields the condition that

$$\mu_i^2 + \mu_j^2 + \mu_k^2 = 1, \qquad i + j + k = (n + 4)/2 \quad , \tag{2}$$

and for any integer q belonging to the set $\{i,j,k\}$, $1 \leq q \leq n/2$.

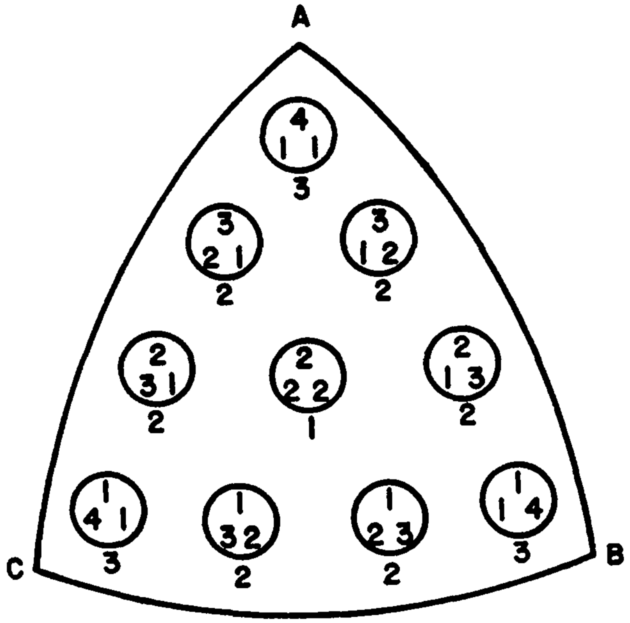

Figure 2. Labeling of octant points for S_8.

Rotation-reflection invariance implies that the points on the octant are equivalent under all permutations of the indices (i,j,k). There are exactly three classes of such points:

A: $i \neq j \neq k$, which occurs six times for each distinct triplet;

B: $i = j \neq k$, $i \neq j = k$, or $i = k \neq j$, which occur three times for each distinct pair; and,

C: $i = j = k$, which occurs only as a center point and, therefore, occurs only once, and only if $(n + 4)/6$ is an integer.

An immediate consequence of rotation-reflection invariance and the law of direction cosines is that

$$\mu_m^2 = \mu_1^2 + (m - 1)D' \quad , \text{ for } m = 1, 2, \ldots, n/2 \quad , \tag{3}$$

and

$$D' = [2/(n - 2)] \, [1 - 3\mu_1^2] \quad . \tag{4}$$

This result may be seen by the following argument: Equation (1) is linear in the variable μ^2. If one picks and fixes any two indices of (i,j,k), then the third index, say k, is determined completely by the constraint that $i + j + k = (n + 4)/2$. Thus, if we pick the third index to be $k + 1$, we have

$$\mu_i^2 + \mu_{j-1}^2 + \mu_{k+1}^2 = 1 \quad , \tag{5}$$

and

$$\mu_{i-1}^2 + \mu_j^2 + \mu_{k+1}^2 = 1 \quad , \tag{6}$$

where

$$i + j + k = (n + 4)/2 \quad ,$$

k is arbitrary in the range $0 \leq k \leq (n - 2)/2$, and $\{i, j\}$ are variable subject to the constraint $i + j = -k + (n + 4)/2$. Subtracting Eq. (6) from Eq. (5), we obtain

$$\mu_i^2 - \mu_{i-1}^2 = \mu_j^2 - \mu_{j-1}^2 \quad , \quad i + j = -k + (n + 4)/2 \quad , \tag{7}$$

which must hold for all $\{i,j\}$ and arbitrary k subject to the specified constraint. From the permutation of labels giving a rotation-reflection invariant description, we can, therefore, conclude that

$$D' = \mu_m^2 - \mu_{m-1}^2 \quad , \quad m = 2, 3, \ldots., n/2 \tag{8}$$

where D' depends only on n. Thus, using transfinite induction we are led to Eq. (3), where n is one of the indices $\{i,j,k\}$ and μ_1^2 is an arbitrary parameter. Finally, substituting Eq. (3) into Eq. (1) yields Eq. (4). If

μ_1^2 is large, the points are clustered toward the center of the octant, for a given n, whereas, if μ_1^2 is small, the points are spread toward the vertices. The μ_1^2 must satisfy the constraint $0 \leq \mu_1^2 \leq 1/3$, since otherwise D', Eq. (4), will not be a positive quantity as assumed. Regardless of the explicit value chosen for μ_1^2, the generated points will then satisfy rotation-reflection invariance. Of course, the generated point weights must satisfy the same symmetries, and they should be nonnegative in order to avoid unphysical results.

It is readily shown that the so-called diffusion theory condition

$$\sum_1^2 p_s Q_s^2 = \sum_1^{n/2} w_m \mu_m^2 = 1/3 \tag{9}$$

is satisfied if the normalization condition is given by

$$\sum_1^S n_s Q_s^2 = \sum_1^{n/2} w_m = 1 \quad , \tag{10}$$

which corresponds to integrating over the octant with the areas expressed in units or $\pi/2$. In Eq. (10) n_s is the multiplicity of points with index s, and S is the total number of points on the octant. For Class A points, $n_s = 6$; for Class B, $n_s = 3$; and for Class C, $n_s = 1$. The other terms in Eq. (9) will be discussed and determined in the next section.

III. AREA METHOD: POINT AND LEVEL WEIGHTS

In order to develop an integration scheme over the octant, we define a discrete set of nonoverlapping areas $\{A_s\}$ covering the octant. We require that the $\{A_s\}$ satisfy the rotation-reflection invariance conditions and that the point weights $\{p_s\}$ are in one-one correspondence with the $\{A_s\}$. On the unit sphere, we represent the integral of N(Q) over dQ, or equivalently over a surface element a, as

$$\int N(Q)dQ = \int N(a)da = \sum \quad p_{\{A_s\}} N_{\{A_s\}}(Q_s) \tag{11}$$

where $N(Q_s)$ is some appropriate representation of $N(Q)$ in dQ_s about Q_s and the $p_{\{A\}}$ are the point weights associated with A_s. The integrals are over the quadrant; the sums are over all the area points.

A variety of assumptions on the connection between the point weights, p_s, and the areas, A_s, are possible, but a simple assumption is that

$$p_s = A_s \tag{12}$$

where the areas have been expressed in units of $\pi/2$.

The S_n point weights, p_s, and the level weights, w_m, can now be determined using the area method. We redefine the point notation to include the insertion of two additional sets of direction cosines, using linear interpolation in μ^2. If we denote these inserted direction cosines by

$$\bar{\mu}_m^2 = \mu_1^2 + (m - 2/3)D \quad , \tag{13}$$

$$\bar{\bar{\mu}}_m^2 = \mu_1^2 + (m - 1/3)D \quad , \tag{14}$$

then the set points defined by Eqs. (3), (13), and (14) can be represented by the relationship

$$\mu_m^2 = \mu_1^2 + (m - 1)D \quad , \quad m = 1, 2, \ldots, \; 1 + 3(n - 2)/2 \tag{15}$$

and

$$D = 2(1 - 3\mu_1^2)/[3(n - 2)] \quad , \quad \mu_1^2 \le 2/3n \quad , \tag{16}$$

where we have defined $\mu_0 = 0$, $\mu_{3n/2} = 1$, and the index constraint has become $i + j + k = 3n/2$. This arrangement of levels and points is illustrated in Fig. 3. Using this extended set of points on the surface

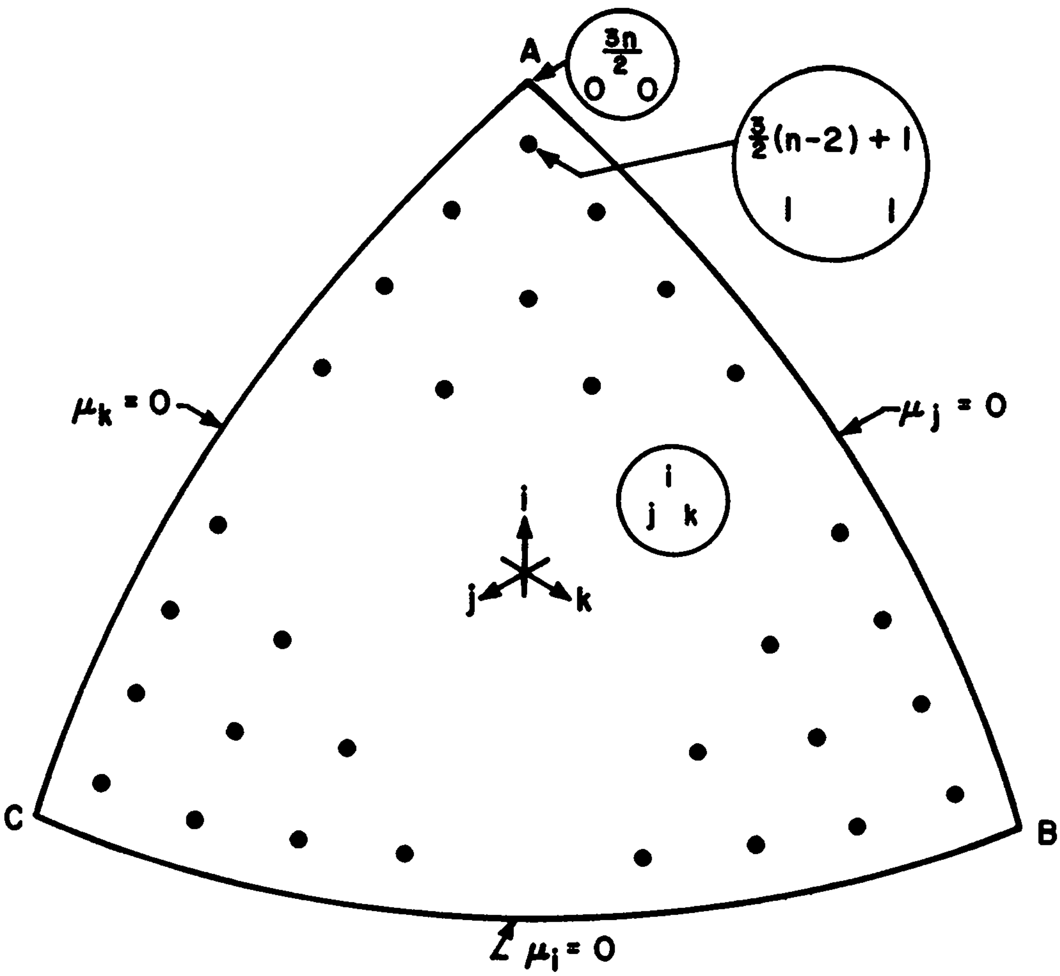

Figure 3. Arrangement of quadrature points on the octant for the area method direction sets.

of the sphere essentially results in a set of "midpoints" for all "triangles" formed by the principal points, μ_m^2 , as shown schematically in projection in Fig. 4. The auxiliary points are then connected by great circle segments or their extensions. Principal points are indicated by the small circles, auxiliary points by dots, as in an example for n = 8 given in Fig. 5.

For the general principal points, $Q_s = (\mu_i, \mu_j, \mu_k)$, a point is called an interior point if $1 \leq m \leq (3n - 2)/2$, where m belongs to {i,j, k}. A point is on the octant boundary if m = 0 or m = 3n/2. Corner points are those for which two indices equal zero, and the third index is 3n/2.

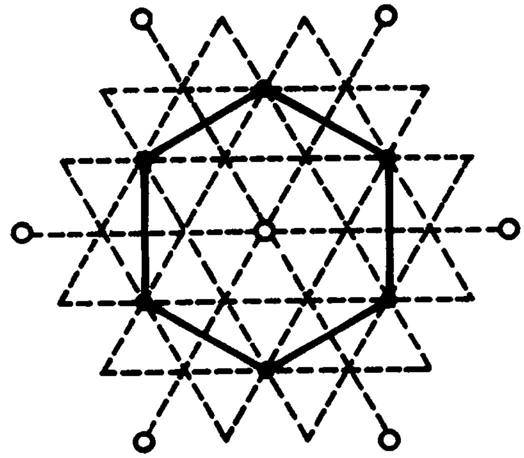

Figure 4. Projection of u_m^2.

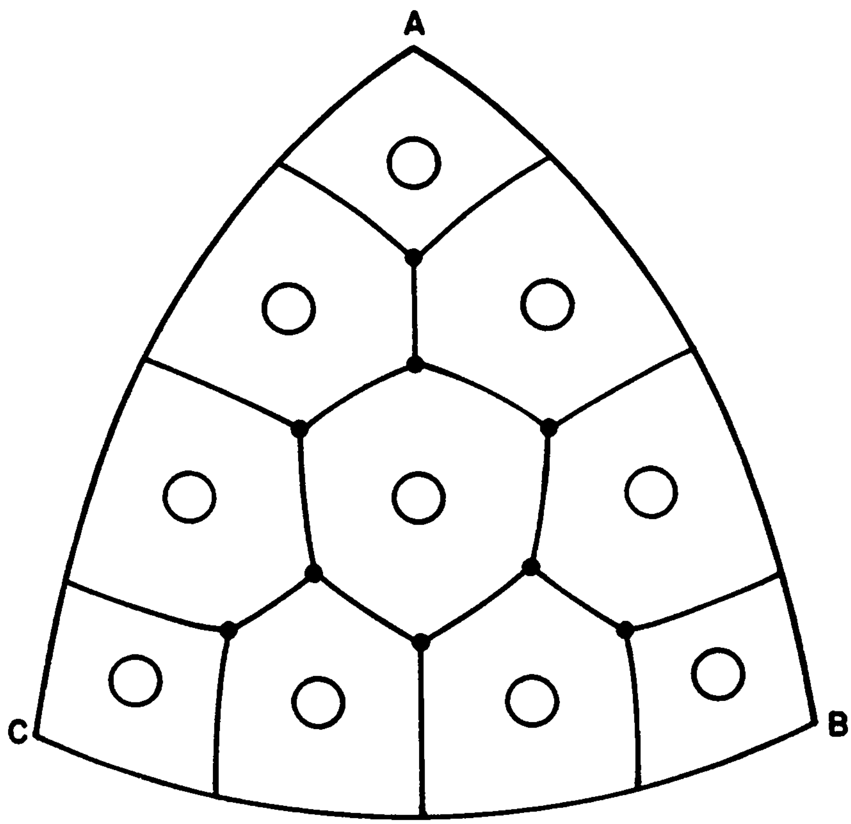

Figure 5. S_8 area domains on the octant

As is readily deduced, the generated figures are spherical hexagons in the central region, spherical pentagons along the boundaries, and spherical quadrilaterals at the vertices. This is illustrated in Fig. 5. The area of the K-sided spherical polygon (in $\pi/2$ units) on the unit sphere ($R^2 = 1$) is given by

$$\text{Area} = (2/\pi)\tilde{A} = \psi - 2(K - 2) \quad , \tag{17}$$

$$\psi = (2/\pi)\tau \quad , \tag{18}$$

where θ is the sum of the interior angles. The expressions for the areas of hexagons ($K = 6$), pentagons ($K = 5$), and quadrilaterals ($K = 4$) are then given by

$$\text{Area}_{\text{hexagon}} = \sum_1^6 a_m - 8 \quad , \tag{19}$$

$$\text{Area}_{\text{hexagon}} = \sum_1^5 a_m - 6 \quad , \tag{20}$$

$$\text{Area}_{\text{quadrilateral}} = \sum_1^1 a_m - 4 \quad , \tag{21}$$

where the a_m are the interior angles illustrated in Figs. 6-8.

Angles are computed using the spherical cosine law

$$\cos(a) = \cos(b)\cos(c) + \sin(b)\sin(c)\cos(A) \quad , \tag{22}$$

where A is the interior angle opposite the arc a and bounded by the arcs b and c, shown in Fig 9.

If we denote the polygon sides as $S_{h_1}, {}_{h_2}$, illustrated in Figs. 6-8, and use Eq. (22) to define the normalized angle function,

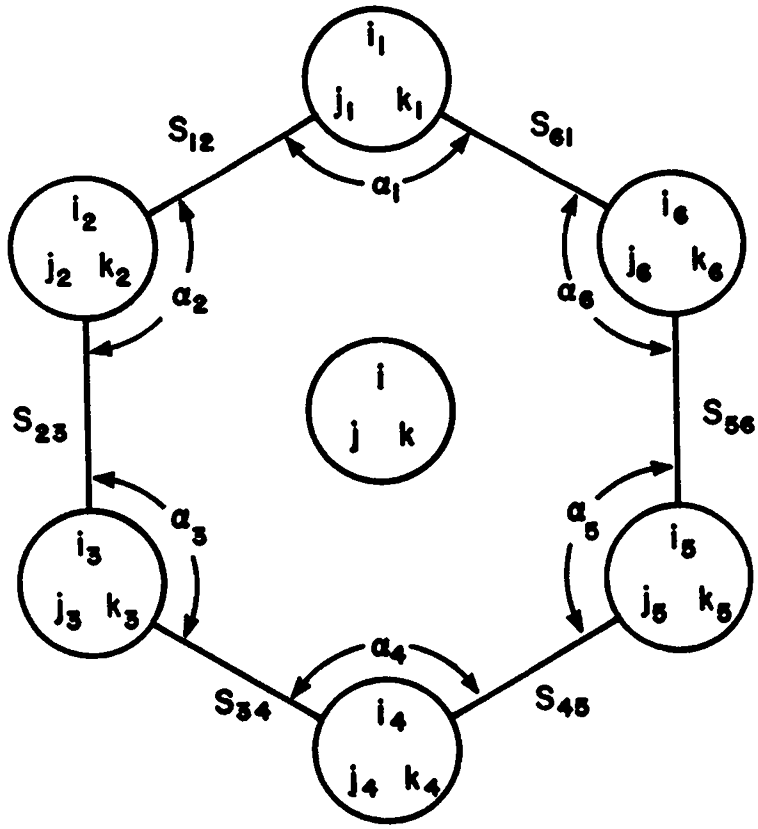

Figure 6. Hexagon spherical geometry labeling.

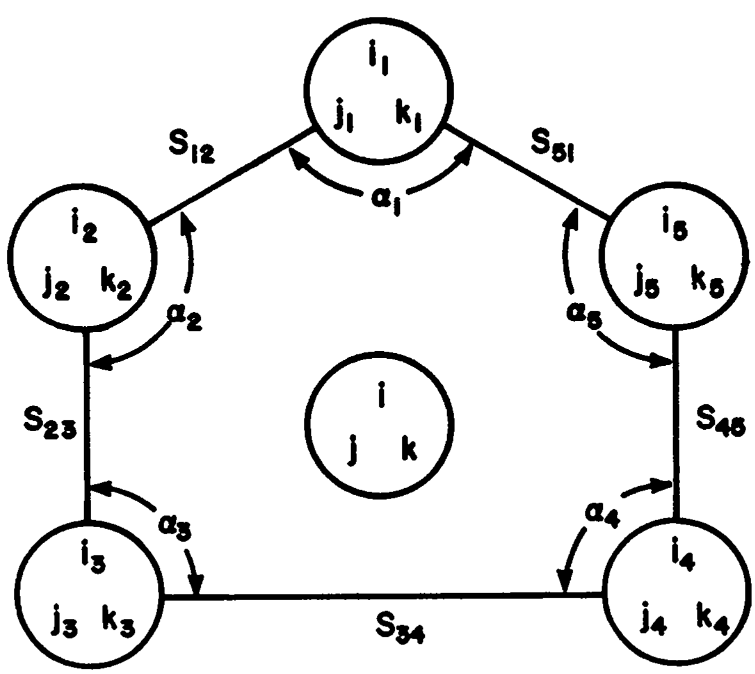

Figure 7. Pentagon spherical geometry labeling.

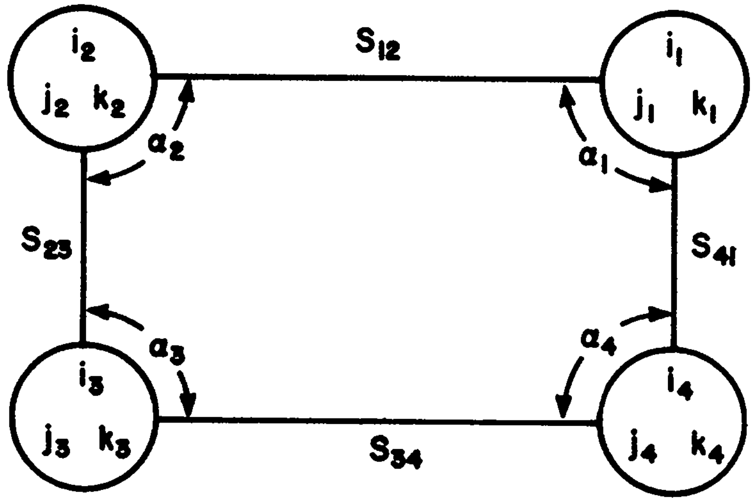

Figure 8. Quadrilateral spherical geometry labeling.

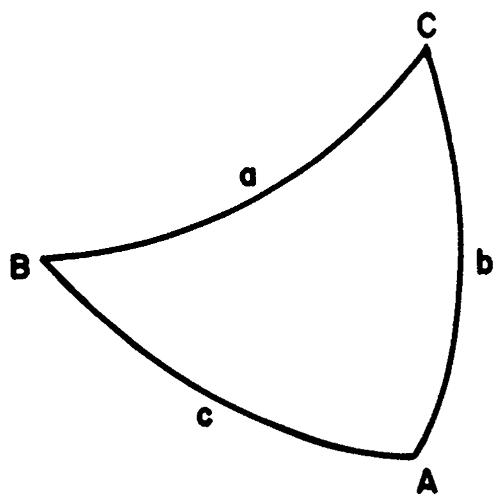

Figure 9. Spherical triangle labeling.

$$a(x,y,z) = (2/\pi) \text{ arc cos}(xy - z)/[(1 - x^2)^{1/2} \ (1 - y^2)]^{1/2}, \tag{23}$$

then the interior angles, a_p are given by

$$ap = a(S_{p_1,p}, S_{p_2,p}, S_{p_1,p_2}) \quad . \tag{24}$$

In Table 1 we list the associated p, p_1, and p_2 values for hexagons, pentagons, and quadrilaterals. In order to complete the calculational prescription, we must identify the subscripts in S_{m_1, m_2}.

TABLE 1

p, p_1, p_2, ASSOCIATIONS

Hexagons

p	1	2	3	4	5	6
p_1	6	1	2	3	4	5
p_2	2	3	4	5	6	1

Pentagons

p	1	2	3	4	5
p_1	5	1	2	3	4
p_2	2	3	4	5	1

Quadrilaterals

p	1	2	3	4
p_1	4	1	2	3
p_2	2	3	4	1

For Hexagons Corresponding (i_m, j_m, k_m) are given in table 2.

TABLE 2

THE HEXAGON (i_m, j_m, k_m) VALUES

m	1	2	3	4	5	6
i_m	$i + 2$	$i + 1$	$i - 1$	$i - 2$	$i - 1$	$i + 1$
j_m	$j - 1$	$j + 1$	$j + 2$	$j + 1$	$j - 1$	$j - 2$
k_m	$k - 1$	$k - 2$	$k - 1$	$k + 1$	$k + 2$	$k + 1$

For pentagons, which occur only if $i = 1$ or $j = 1$, or $k = 1$, along the octant edges, there are three distinct cases, which we summarize in Tables 3 and 4.

TABLE 3

THE HEXAGON (i_m, j_m, k_m) VALUES

m	1	2	3	4	5
i_m	$i + x_1^1$	$i + x_2^1$	$i + x_3^1$	$i + x_4^1$	$i + x_5^1$
j_m	$j + x_1^2$	$j + x_2^2$	$j + x_3^2$	$j + x_4^2$	$j + x_5^2$
k_m	$k + x_1^3$	$j + x_2^3$	$k + x_3^3$	$j + x_4^3$	$j + x_5^3$

TABLE 4

THE PENTAGON x_m^n ($i = 1$)

m	1	2	3	4	5
x_m^1	2	1	-1	-1	1
x_m^2	-1	1	2	-1	-2
x_m^3	-1	-2	-1	2	1

For $i = 1$, we use (x_m^1, x_m^2, x_m^3,) in Table 4. For $j = 1$, we use (x_m^2, x_m^3, x_m^1), and for $k = 1$, we use (x_m^3, x_m^1, x_m^2).

Finally, for quadrilaterals, which occur only if $i = j = 1$, if $j = k = 1$, or if $k = i = 1$ at the octant vertices, there are again three distinct cases, which are summarized in Tables 5 and 6.

TABLE 5

THE QUADRILATERAL (i_m,j_m,k_m) VALUES

m	1	2	3	4
i_m	$i + x_1^1$	$i + x_2^1$	$i + x_3^1$	$i + x_4^1$
j_m	$j + x_1^2$	$j + x_2^2$	$j + x_3^2$	$j + x_4^2$
k_m	$k + x_1^3$	$k + x_2^3$	$k + x_3^3$	$j + x_4^3$

TABLE 6

THE QUADRILATERAL x_m^n $(i = j = 1)$

m	1	2	3	4
x_m^1	2	1	-1	-1
x_m^2	-1	1	2	-1
x_m^3	-1	-2	-1	2

For $i = j = 1$, we use (x_m^1, x_m^2, x_m^3), for the column in Table 6. For $i = k = 1$, we use (x_m^2, x_m^3, x_m^1), and for $j = k = 1$, we use (x_m^3, x_m^1, x_m^2).

IV. CLASSIFICATION OF FIGURES

Three classes of points (A, B, and C) and three classes of figures (hexagons, pentagons, and quadrilaterals) occur in the area method. Every point is covered by a figure and every figure contains a point. An enumeration of points and figures on the octant indicates that, percentage-wise, in the limit of large n, almost all the figures are of the type Class A hexagons with octant multiplicity $n_s - 6$.

If we denote the indices of an octant point by (i,j,k), the point classes and their associated octant multiplicities (n_s) are

- Class A points -- $i \neq j \neq k$, $n_s = 6$;
- Class B points -- two indices of (i,j,k) equal, $n_s = 3$; and
- Class C points -- $i = j = k$, $n_s = 1$, but this occurs only if $i = (n + 4)/6$ is integer.

The total number of distinct figures is given by

$$S = N(iii) + N(iik) + N(ijk) \quad , \tag{25}$$

where N(iii) is the number of Class C (0 or 1), N(iik) is the number in Class B, and N(ijk) is the number in Class A.

If we use the fact that

$$\sum_{1}^{n/2} k = n(n + 2)/8 \tag{26}$$

and count the octant multiplicities, it is clear that the total number of points on the octant in n/2 triangular levels is

$$P = N(iii) + 3N(iik) + 6N(ijk). \tag{27}$$

By direct enumeration it is easily shown that the total number of points in the Classes A, B, and C is given by (m = 0, 1,2, ...).

$$N(iii) = \begin{cases} 0 \text{ if } n = 6m + 2 \\ 1 \text{ if } n = 6m + 2 \end{cases}$$

$$N(iik) = \begin{cases} n/4 \text{ if } n/2 \text{ is even and } n = 6m + 2 \\ (n + 2)/4 \text{ if } n/2 \text{ is odd and } n = 6m + 2 \\ (n - 1)/4 \text{ if } n/2 \text{ is even and } n = 6m + 2 \\ (n - 2)/4 \text{ if } n/2 \text{ is odd and } n = 6m + 2 \end{cases}$$

$$N(ijk) = \begin{cases} n(n - 4)/48 \text{ if } n/2 \text{ is even and } n = 6m + 2 \\ (n + 2)(n - 6)/48 \text{ if } n/2 \text{ is odd and } n = 6m + 2 \\ (n^2 - 4n + 16)/48 \text{ if } n/2 \text{ is even and } n = 6m + 2 \\ (n - 2)^2/48 \text{ if } n/2 \text{ is odd and } n = 6m + 2 \quad . \end{cases} \tag{28}$$

If we use Eq. (28), the total number of distinct figures is (m = 0, 1, 2, ...)

$$S = \begin{cases} n(n + 8)/48 \text{ if } n/2 \text{ is even, and } n = 6m + 2 \\ (n + 2)(n + 6)/48 \text{ if } n/2 \text{ is odd and } n = 6m + 2 \\ (n + 4)^2/48 \text{ if } n/2 \text{ is even and } n = 6m + 2 \\ (n^2 + 8n + 28)/48 \text{ if } n/2 \text{ is odd and } n = 6m + 2. \end{cases} \quad (29)$$

From these formulae, construction of Tables for N(iii), N(iik), and N(ijk), and S for various n can be made.[3] From Eq. (28), we see that the limiting values for large n take the form

$$N\,(iii) = 0 \text{ or } 1$$

$$N\,(iik) \simeq n/4$$

$$N\,(ijk) \simeq n^2/48 \quad (30)$$

so that, percentage-wise for large n, almost all the figures occur with three unequal indices.

The N(iik) points lie in hexagons, pentagons, and quadrilaterals. The N(ijk) points lie in hexagons and pentagons. The N(iii) points lie only in hexagons. Enumerating the number of distinct figures f of a given class and type (h = hexagon, p = pentagon, q = quadrilateral), we can easily show that (m = 0, 1, 2, ...).

$$f_q(iii) = 0 \quad \text{all } n/2$$

$$f_p(iii) = 0 \quad \text{all } n/2$$

$$f_h(iii) = \begin{cases} 0 & \text{if } n = 6m + 2 \\ 1 & \text{if } n = 6m + 2 \end{cases}$$

$$f_q(iik) = 1 \quad \text{all } n/2$$

$$f_p(iik) = \begin{cases} 0 & \text{if } n/2 \text{ is even} \\ 1 & \text{if } n/2 \text{ is odd} \end{cases}$$

$$f_h(iik) = \begin{cases} (n-4)/4 \text{ if } n/2 \text{ is even and } n = 6m+2 \\ (n-8)/4 \text{ if } n/2 \text{ is even and } n = 6m+2 \\ (n-6)/4 \text{ if } n/2 \text{ is odd and } n = 6m+2 \\ (n-10)/4 \text{ if } n/2 \text{ is odd and } n = 6m+2 \end{cases}$$

and

$$f_q(kjk) = 0 \quad \text{all } n/2$$

$$f_p(ijk) = \begin{cases} (n-4)/4 \text{ if } 2 \text{ is even} \\ (n-12)/8 \text{ if } n/2 \text{ is odd} \end{cases}$$

$$f_h(ijk) = \begin{cases} (n-4)(n-12)/48 \text{ if } n/2 \text{ is even } n = 6m+2 \\ (n-6)(n-10)/48 \text{ if } n/2 \text{ is odd and } n = 6m+2 \\ (n-8)^2/48 \text{ if } n/2 \text{ is odd and } n = 6m+2 \\ (n^2-16n+76)/48 \text{ if } n/2 \text{ is odd and } n = 6m+2 \end{cases} \quad .(31)$$

From the definitions of the N's and f's, we have

$$\begin{aligned} N(iii) &= f_q(iii) + f_p(iii) + f_h(iii) \\ N(iii) &= f_q(iik) + f_p(iik) + f_h(iik) \\ N(iii) &= f_q(ijk) + f_p(ijk) + f_h(ijk) \quad . \end{aligned} \qquad (32)$$

Defining g as the total number of figures of a given type, we have

$$\begin{aligned} g_q &= f_q(iii) + f_q(iik) + f_q(ijk) \\ g_p &= g_p(iii) + g_p(iik) + g_p(ijk) \\ g_h &= g_h(iii) + g_h(iik) + g_p(ijk) \end{aligned} \qquad (33)$$

where, of course,

$$S = g_q + g_p + g_h = N(iii) + N(iik) + N(ijk) \quad . \qquad (34)$$

Then, the g's are given by

$$g_q = 1 \text{ for all } n/2$$

$$g_p = \begin{cases} (n-4)/4 \text{ if } n/2 \text{ is even} \\ (n-2)/4 \text{ if } n/2 \text{ is odd} \end{cases}$$

$$g_h = \begin{cases} n(n-4)/48 \text{ if } n/2 \text{ is even and } n = 6m+2 \\ (n+2)(n-6)/48 \text{ if } n/2 \text{ is odd and } n = 6m+2 \\ (n^2-4n+16)/48 \text{ if } n/2 \text{ is even and } n = 6m+2 \\ (n^2-4n+4)/48 \text{ if } n/2 \text{ is odd and } n = 6m+2 \end{cases} \quad . \qquad (35)$$

It is interesting to note that

$$g_h = N(ijk)$$

$$g_q + g_p = N(iii) + N(iik) \quad ; \qquad (36)$$

that is, the total number of hexagons is always the same as the number of figures occurring six times, even though hexagons occur with octant multiplicity $n_g = 1$, 3, and 6. Similarly, the total number of quadrilaterals and pentagons is the same as the figures occurring one and three times. Finally, it is readily seen that in the limit of large n, almost all the figures are hexagons.

V. AREA METHOD DIRECTION SETS

Using the area method techniques described in the previous two sections, a computer program for generating the direction sets was written in FORTRAN for usage on the CDC-7600. The program constructs the point weights and directions as used by the one- and two-dimensional S_n transport codes.

Using this program as a subroutine, we can construct direction sets to full machine significance at execution time rather than relying on transcriptions that can be prone to error. It is now possible to generate automatically direction sets for much larger n previously available for a specific choice of μ_1^2, if needed.

The point weights and directions were evaluated in single precision on the CDC-7600 and are positive for all even n less than 2000. In double precision, they are positive for all even n less than 10 000, which is the largest case that was constructed and examined in the testing. Comparison of the single- and double-precision results indicated differences in the 12th digit for n greater than 100. The preprocessing code renormalizes the sum of the weights to unity at run time.

These direction sets differ from those produced previously using the asymptotic directions[2,3] for two reasons: (1) An implicit assumption on the orthogonality of intersecting level lines with quadrant boundaries was incorrect and led to a negative area calculation for high order n; and, (2) the free parameter, μ_1^2, given by

$$\mu_1^2 = 1/[3(n - 1)] \tag{37}$$

was used previously. In this investigation the parameter μ_1, given by

$$\mu_1 = 0.69/n^{1/2} - 0.001 \quad , \tag{38}$$

was used. Comparison of the level moment conditions

$$Q_{k,n} = \sum_1^{n/2} W_m u_m^k - 1/(k + 1) \tag{39}$$

indicates that the current choice of μ_1^2, Eq. (38), gives level moments by a factor of 2 better than asymptotic choice, Eq. (37), which was used previously. Of course, neither of these choices for μ_1^2 yield level moments as accurate by several orders of magnitude as do the weights and direction of the P_{n-1} and $DP_{(n-2)/2}$ methods, which were constructed specifically to satisfy the moments exactly. However, the P_{n-1} and $DP_{(n-2)/2}$ direction sets do not generalize to cylindrical one-dimensional geometry or to multidimensional S_n quadrature on the unit sphere, which was the origin of this problem over two decades ago.

In Table 7, we list the asymptotic S_n directions for slabs, cylinders, and spheres constructed by the computer program using μ_1^2 given by Eq. (37). In Table 8, we list the area method direction sets for

TABLE 7

ASYMPTOTIC DIRECTION (AD) WEIGHTS AND DIRECTIONS

N = 2 IGE =1 SLABS AND SPHERES

W(1)=	0.000000000000000	UB(1)=	-1.000000000000000
W(2)=	.500000000000000	UB(2)=	-.577350269189626
W(3)=	.500000000000000	UB(3)=	.577350269189626

MOMENTS

N = 2 Q(1)= 0.

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	15.4701	0.0000	-23.0200	-44.4444

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS = 16.5869

N = 2 IGE =2 CYLINDERS

W(1)=	0.000000000000000	UB(1)=	-.816496580927726
W(2)=	.500000000000000	UB(2)=	-.577350269189626
W(3)=	.500000000000000	UB(3)=	.577350269189626

N = 4 IGE =1 SLABS AND SPHERES

W(1)=	0.000000000000000	UB(1)=	-1.000000000000000
W(2)=	.166666666666666	UB(2)=	-.881917103688192
W(3)=	.333333333333332	UB(3)=	-.333333333333332
W(4)=	.333333333333332	UB(4)=	.333333333333332
W(5)=	.166666666666666	UB(5)=	.881917103688192

MOMENTS

N = 4 Q(1)= 0.

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	3.2389	1.3346	8.3472	10.8968
2	-.0000	4.9383	10.4252	9.8765

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS = 8.4508

N = 4 IGE =2 CYLINDERS

W(1)=	0.000000000000000	UB(1)=	-.942809041582063
W(2)=	.166666666666666	UB(2)=	-.881917103688192
W(3)=	.166666666666666	UB(3)=	-.333333333333332
W(4)=	.166666666666666	UB(4)=	.333333333333332
W(5)=	.166666666666666	UB(5)=	.881917103688192
W(6)=	0.000000000000000	UB(6)=	-.471404520791038
W(7)=	.166666666666666	UB(7)=	-.333333333333332
W(8)=	.166666666666666	UB(8)=	.333333333333332

Table 7 (continued)

N = 6   IGE =1   SLABS AND SPHERES

W(1)=	0.000000000000000	UB(1)=	-1.000000000000000
W(2)=	.100451046290544	UB(2)=	-.930949336251260
W(3)=	.132431240752243	UB(3)=	-.683130051063969
W(4)=	.267117712957210	UB(4)=	-.258198889747160
W(5)=	.267117712957210	UB(5)=	.258198889747160
W(6)=	.132431240752243	UB(6)=	.683130051063969
W(7)=	.100451046290544	UB(7)=	.930949336251260

MOMENTS

N =   6   Q(1)= 0.

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	1.1808	5.4777	12.1429	14.5023
2	-.0000	8.2981	13.3233	14.5425
3	2.2900	10.4992	14.1001	14.2291

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS =   8.5066

N = 6   IGE =2   CYLINDERS

W(1)=	0.000000000000000	UB(1)=	-.966091783079293
W(2)=	.100451046290544	UB(2)=	-.930949336251260
W(3)=	.066215620376122	UB(3)=	-.683130051063969
W(4)=	.100451046290544	UB(4)=	-.258198889747160
W(5)=	.100451046290544	UB(5)=	.258198889747160
W(6)=	.066215620376122	UB(6)=	.683130051063969
W(7)=	.100451046290544	UB(7)=	.930949336251260
W(8)=	0.000000000000000	UB(8)=	-.730296743340226
W(9)=	.066215620376122	UB(9)=	-.683130051063969
W(10)=	.066215620376122	UB(10)=	-.258198889747160
W(11)=	.066215620376122	UB(11)=	.258198889747160
W(12)=	.066215620376122	UB(12)=	.683130051063969
W(13)=	0.000000000000000	UB(13)=	-.365148371670120
W(14)=	.100451046290544	UB(14)=	-.258198889747160
W(15)=	.100451046290544	UB(15)=	.258198889747160

N = 8   IGE =1   SLABS AND SPHERES

W(1)=	0.000000000000000	UB(1)=	-1.000000000000000
W(2)=	.072242022223608	UB(2)=	-.951189731211340
W(3)=	.085280409813437	UB(3)=	-.786795792469441
W(4)=	.112713113702299	UB(4)=	-.577350269189626
W(5)=	.229764454260654	UB(5)=	-.218217890235993
W(6)=	.229764454260654	UB(6)=	.218217890235993
W(7)=	.112713113702299	UB(7)=	.577350269189626
W(8)=	.085280409813437	UB(8)=	.786795792469441
W(9)=	.072242022223608	UB(9)=	.951189731211340

Table 7 (continued)

MOMENTS

N = 8 Q(1)= 0.

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	.4111	7.1697	13.4076	16.3643
2	-.0000	9.1095	14.4343	16.6280
3	2.2302	10.7626	15.2689	16.7065
4	4.8627	12.1866	15.9119	16.6037

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS = 10.1210

N = 8 IGE =2 CYLINDERS

W(1)=	0.000000000000000	UB(1)=	-.975900072948530
W(2)=	.072242022223608	UB(2)=	-.951189731211340
W(3)=	.042640204906719	UB(3)=	-.786795792469441
W(4)=	.042640204906719	UB(4)=	-.577350269189626
W(5)=	.072242022223608	UB(5)=	-.218217890235993
W(6)=	.072242022223608	UB(6)=	.218217890235993
W(7)=	.042640204906719	UB(7)=	.577350269189626
W(8)=	.042640204906719	UB(8)=	.786795792469441
W(9)=	.072242022223608	UB(9)=	.951189731211340
W(10)=	0.000000000000000	UB(10)=	-.816496580927726
W(11)=	.042640204906719	UB(11)=	-.786795792469441
W(12)=	.027432703888862	UB(12)=	-.577350269189626
W(13)=	.042640204906719	UB(13)=	-.218217890235993
W(14)=	.042640204906719	UB(14)=	.218217890235993
W(15)=	.027432703888862	UB(15)=	.577350269189626
W(16)=	.042640204906719	UB(16)=	.786795792469441
W(17)=	0.000000000000000	UB(17)=	-.617213399848371
W(18)=	.042640204906719	UB(18)=	-.577350269189626
W(19)=	.042640204906719	UB(19)=	-.218217890235993
W(20)=	.042640204906719	UB(20)=	.218217890235993
W(21)=	.042640204906719	UB(21)=	.577350269189626
W(22)=	0.000000000000000	UB(22)=	-.308606699924184
W(23)=	.072242022223608	UB(23)=	-.218217890235993
W(24)=	.072242022223608	UB(24)=	.218217890235993

N =10 IGE =1 SLABS AND SPHERES

W(1)=	0.000000000000000	UB(1)=	-1.000000000000000
W(2)=	.056462466014014	UB(2)=	-.962250448649375
W(3)=	.063585277886588	UB(3)=	-.838870492807857
W(4)=	.074919355157779	UB(4)=	-.693888666488711
W(5)=	.100222258635287	UB(5)=	-.509175077217314
W(6)=	.204810642306331	UB(6)=	-.192450089729875
W(7)=	.204810642306331	UB(7)=	.192450089729875
W(8)=	.100222258635287	UB(8)=	.509175077217314
W(9)=	.074919355157779	UB(9)=	.693888666488711
W(10)=	.063585277886588	UB(10)=	.838870492807857
W(11)=	.056462466014014	UB(11)=	.962250448649375

Table 7 (continued)

MOMENTS

N = 10 Q(1)= -.7105E-12

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	.0412	7.9384	14.2070	17.4597
2	-.0000	9.4578	15.0920	17.7635
3	2.0497	10.8326	15.8589	17.9547
4	4.2804	12.0790	16.5086	18.0350
5	6.2355	13.2031	17.0418	18.0066

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS = 11.1450

N =10 IGE =2 CYLINDERS

W(1)=	0.000000000000000	UB(1)=	-.981304762925314
W(2)=	.056462466014014	UB(2)=	-.962250448649375
W(3)=	.031792638943294	UB(3)=	-.838870492807857
W(4)=	.028300432391715	UB(4)=	-.693888466488711
W(5)=	.031792638943294	UB(5)=	-.509175077217314
W(6)=	.056462466014014	UB(6)=	-.192450089729875
W(7)=	.056462466014014	UB(7)=	.192450089729875
W(8)=	.031792638943294	UB(8)=	.509175077217314
W(9)=	.028300432391715	UB(9)=	.693888466488711
W(10)=	.031792638943294	UB(10)=	.838870492807857
W(11)=	.056462466014014	UB(11)=	.962250448649375
W(12)=	0.000000000000000	UB(12)=	-.860662065823870
W(13)=	.031792638943294	UB(13)=	-.838870492807857
W(14)=	.018318490374349	UB(14)=	-.693888466488711
W(15)=	.018318490374349	UB(15)=	-.509175077217314
W(16)=	.031792638943294	UB(16)=	-.192450089729875
W(17)=	.031792638943294	UB(17)=	.192450089729875
W(18)=	.018318490374349	UB(18)=	.509175077217314
W(19)=	.018318490374349	UB(19)=	.693888466488711
W(20)=	.031792638943294	UB(20)=	.838870492807857
W(21)=	0.000000000000000	UB(21)=	-.720082299823098
W(22)=	.028300432391715	UB(22)=	-.693888466488711
W(23)=	.018318490374349	UB(23)=	-.509175077217314
W(24)=	.028300432391715	UB(24)=	-.192450089729875
W(25)=	.028300432391715	UB(25)=	.192450089729875
W(26)=	.018318490374349	UB(26)=	.509175077217314
W(27)=	.028300432391715	UB(27)=	.693888466488711
W(28)=	0.000000000000000	UB(28)=	-.544331053051826
W(29)=	.031792638943294	UB(29)=	-.509175077217314
W(30)=	.031792638943294	UB(30)=	-.192450089729875
W(31)=	.031792638943294	UB(31)=	.192450089729875
W(32)=	.031792638943294	UB(32)=	.509175077217314
W(33)=	0.000000000000000	UB(33)=	-.272165526975913
W(34)=	.056462466014014	UB(34)=	-.192450089729875
W(35)=	.056462466014014	UB(35)=	.192450089729875

Table 7 (continued)

N =12   IGE =1   SLABS AND SPHERES

W(1)=	0.000000000000000	UB(1)=	-1.000000000000000
W(2)=	.046357556555418	UB(2)=	-.969223369195117
W(3)=	.050838290719928	UB(3)=	-.870388279778485
W(4)=	.057003591715130	UB(4)=	-.758786910639326
W(5)=	.067974072877962	UB(5)=	-.627645914460846
W(6)=	.091233466775218	UB(6)=	-.460566186471835
W(7)=	.186593021356344	UB(7)=	-.174077455955697
W(8)=	.186593021356344	UB(8)=	.174077455955697
W(9)=	.091233466775218	UB(9)=	.460566186471835
W(10)=	.067974072877962	UB(10)=	.627645914460846
W(11)=	.057003591715130	UB(11)=	.758786910639326
W(12)=	.050838290719928	UB(12)=	.870388279778485
W(13)=	.046357556555418	UB(13)=	.969223369195117

MOMENTS

N = 12   Q(1)= -.7105E-12

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	-.1609	8.4021	14.7455	18.1744
2	-.0000	9.6772	15.5137	18.4747
3	1.8701	10.8601	16.2024	18.6993
4	3.8076	11.9559	16.8123	18.8492
5	5.5100	12.9677	17.3439	18.9253
6	7.0226	13.8971	17.7977	18.9290

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS = 11.8639

N =12   IGE =2   CYLINDERS

W(1)=	0.000000000000000	UB(1)=	-.984731927834662
W(2)=	.046357556555418	UB(2)=	-.969223369195117
W(3)=	.025419145359964	UB(3)=	-.870388279778485
W(4)=	.021519808762790	UB(4)=	-.758786910639326
W(5)=	.021519808762790	UB(5)=	-.627645914460846
W(6)=	.025419145359964	UB(6)=	-.460566186471835
W(7)=	.046357556555418	UB(7)=	-.174077455955697
W(8)=	.046357556555418	UB(8)=	.174077455955697
W(9)=	.025419145359964	UB(9)=	.460566186471835
W(10)=	.021519808762790	UB(10)=	.627645914460846
W(11)=	.021519808762790	UB(11)=	.758786910639326
W(12)=	.025419145359964	UB(12)=	.870388279778485
W(13)=	.046357556555418	UB(13)=	.969223369195117
W(14)=	0.000000000000000	UB(14)=	-.887625364598595
W(15)=	.025419145359964	UB(15)=	-.870388279778485
W(16)=	.013963974189550	UB(16)=	-.758786910639326
W(17)=	.012467227676191	UB(17)=	-.627645914460846
W(18)=	.013963974189550	UB(18)=	-.460566186471835
W(19)=	.025419145359964	UB(19)=	-.174077455955697
W(20)=	.025419145359964	UB(20)=	.174077455955697

Table 7 (continued)

W(21)=	.013963974189550	UB(21)=	.460566186471835
W(22)=	.012467227676191	UB(22)=	.627645914460846
W(23)=	.013963974189550	UB(23)=	.758786910639326
W(24)=	.025419145359964	UB(24)=	.870388279778485
W(25)=	0.000000000000000	UB(25)=	-.778493894161527
W(26)=	.021519808762790	UB(26)=	-.758786910639326
W(27)=	.012467227676191	UB(27)=	-.627645914460846
W(28)=	.012467227676191	UB(28)=	-.460566186471835
W(29)=	.021519808762790	UB(29)=	-.174077655955697
W(30)=	.021519808762790	UB(30)=	.174077655955697
W(31)=	.012467227676191	UB(31)=	.460566186471835
W(32)=	.012467227676191	UB(32)=	.627645914460846
W(33)=	.021519808762790	UB(33)=	.758786910639326
W(34)=	0.000000000000000	UB(34)=	-.651338947278933
W(35)=	.021519808762790	UB(35)=	-.627645914460846
W(36)=	.013963974189550	UB(36)=	-.460566186471835
W(37)=	.021519808762790	UB(37)=	-.174077655955697
W(38)=	.021519808762790	UB(38)=	.174077655955697
W(39)=	.013963974189550	UB(39)=	.460566186471835
W(40)=	.021519808762790	UB(40)=	.627645914460846
W(41)=	0.000000000000000	UB(41)=	-.492365963917338
W(42)=	.025419145359964	UB(42)=	-.460566186471835
W(43)=	.025419145359964	UB(43)=	-.174077655955697
W(44)=	.025419145359964	UB(44)=	.174077655955697
W(45)=	.025419145359964	UB(45)=	.460566186471835
W(46)=	0.000000000000000	UB(46)=	-.246182981958669
W(47)=	.046357556555418	UB(47)=	-.174077655955697
W(48)=	.046357556555418	UB(48)=	.174077655955697

N =14 IGE =1 SLABS AND SPHERES

W(1)=	0.000000000000000	UB(1)=	-1.000000000000000
W(2)=	.039326869832498	UB(2)=	-.974021534011413
W(3)=	.042397011687780	UB(3)=	-.891555828241724
W(4)=	.046220061116284	UB(4)=	-.800640769025435
W(5)=	.052449506373218	UB(5)=	-.697982440452112
W(6)=	.062746578425365	UB(6)=	-.577350269189626
W(7)=	.084331802130589	UB(7)=	-.423659272868161
W(8)=	.172528170434266	UB(8)=	-.160128153805087
W(9)=	.172528170434266	UB(9)=	.160128153805087
W(10)=	.084331802130589	UB(10)=	.423659272868161
W(11)=	.062746578425365	UB(11)=	.577350269189626
W(12)=	.052449506373218	UB(12)=	.697982440452112
W(13)=	.046220061116284	UB(13)=	.800640769025435
W(14)=	.042397011687780	UB(14)=	.891555828241724
W(15)=	.039326869832498	UB(15)=	.974021534011413

Table 7 (continued)

MOMENTS

N = 14 Q(1)= -.7105E-12

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	-.2799	8.7296	15.1281	18.6761
2	-.0000	9.8323	15.8043	18.9607
3	1.7118	10.8684	16.4234	19.1910
4	3.4272	11.8408	16.9858	19.3674
5	4.9388	12.7517	17.4920	19.4907
6	6.2990	13.6026	17.9421	19.5615
7	7.5551	14.3944	18.3367	19.5804

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS = 12.3852

N =14 IGE =2 CYLINDERS

W(1)=	0.000000000000000	UB(1)=	-.987096233585646
W(2)=	.039326869832498	UB(2)=	-.974021534011413
W(3)=	.021198505843890	UB(3)=	-.891555828241724
W(4)=	.017442839180850	UB(4)=	-.800640769025435
W(5)=	.016591740719787	UB(5)=	-.697982440452112
W(6)=	.017442839180850	UB(6)=	-.577350269189626
W(7)=	.021198505843890	UB(7)=	-.423659272868161
W(8)=	.039326869832498	UB(8)=	-.160128153805087
W(9)=	.039326869832498	UB(9)=	.160128153805087
W(10)=	.021198505843890	UB(10)=	.423659272868161
W(11)=	.017442839180850	UB(11)=	.577350269189626
W(12)=	.016591740719787	UB(12)=	.697982440452112
W(13)=	.017442839180850	UB(13)=	.800640769025435
W(14)=	.021198505843890	UB(14)=	.891555828241724
W(15)=	.039326869832498	UB(15)=	.974021534011413
W(16)=	0.000000000000000	UB(16)=	-.905821627915675
W(17)=	.021198505843890	UB(17)=	-.891555828241724
W(18)=	.011334382754583	UB(18)=	-.800640769025435
W(19)=	.009633012466822	UB(19)=	-.697982440452112
W(20)=	.009633012466822	UB(20)=	-.577350269189626
W(21)=	.011334382754583	UB(21)=	-.423659272868161
W(22)=	.021198505843890	UB(22)=	-.160128153805087
W(23)=	.021198505843890	UB(23)=	.160128153805087
W(24)=	.011334382754583	UB(24)=	.423659272868161
W(25)=	.009633012466822	UB(25)=	.577350269189626
W(26)=	.009633012466822	UB(26)=	.697982440452112
W(27)=	.011334382754583	UB(27)=	.800640769025435
W(28)=	.021198505843890	UB(28)=	.891555828241724
W(29)=	0.000000000000000	UB(29)=	-.816496580927726
W(30)=	.017442839180850	UB(30)=	-.800640769025435

Table 7 (continued)

W(31)=	.009633012466822	UB(31)=	-.697982440452112
W(32)=	.008594875130020	UB(32)=	-.577350269189626
W(33)=	.009633012466822	UB(33)=	-.423659272868161
W(34)=	.017442839180850	UB(34)=	-.160128153805087
W(35)=	.017442839180850	UB(35)=	.160128153805087
W(36)=	.009633012466822	UB(36)=	.423659272868161
W(37)=	.008594875130020	UB(37)=	.577350269189626
W(38)=	.009633012466822	UB(38)=	.697982440452112
W(39)=	.017442839180850	UB(39)=	.800640769025435
W(40)=	0.000000000000000	UB(40)=	-.716114874039430
W(41)=	.016591740719787	UB(41)=	-.697982440452112
W(42)=	.009633012466822	UB(42)=	-.577350269189626
W(43)=	.009633012466822	UB(43)=	-.423659272868161
W(44)=	.016591740719787	UB(44)=	-.160128153805087
W(45)=	.016591740719787	UB(45)=	.160128153805087
W(46)=	.009633012466822	UB(46)=	.423659272868161
W(47)=	.009633012466822	UB(47)=	.577350269189626
W(48)=	.016591740719787	UB(48)=	.697982440452112
W(49)=	0.000000000000000	UB(49)=	-.599144689815278
W(50)=	.017442839180850	UB(50)=	-.577350269189626
W(51)=	.011334382754583	UB(51)=	-.423659272868161
W(52)=	.017442839180850	UB(52)=	-.160128153805087
W(53)=	.017442839180850	UB(53)=	.160128153805087
W(54)=	.011334382754583	UB(54)=	.423659272868161
W(55)=	.017442839180850	UB(55)=	.577350269189626
W(56)=	0.000000000000000	UB(56)=	-.452910813657846
W(57)=	.021198505843890	UB(57)=	-.423659272868161
W(58)=	.021198505843890	UB(58)=	-.160128153805087
W(59)=	.021198505843890	UB(59)=	.160128153805087
W(60)=	.021198505843890	UB(60)=	.423659272868161
W(61)=	0.000000000000000	UB(61)=	-.226455406828919
W(62)=	.039326869832498	UB(62)=	-.160128153805087
W(63)=	.039326869832498	UB(63)=	.160128153805087

N =16 IGE =1 SLABS AND SPHERES

W(1)=	0.000000000000000	UB(1)=	-1.000000000000000
W(2)=	.034150743173670	UB(2)=	-.977525219907672
W(3)=	.036379491801739	UB(3)=	-.906764700582357
W(4)=	.038944135405336	UB(4)=	-.829993306532575
W(5)=	.042967202111948	UB(5)=	-.745355992499928
W(6)=	.048915735831746	UB(6)=	-.649786289653928
W(7)=	.058595584986041	UB(7)=	-.537483849886566
W(8)=	.078806650698751	UB(8)=	-.394405318873305
W(9)=	.161240455990769	UB(9)=	-.149071198499986
W(10)=	.161240455990769	UB(10)=	.149071198499986
W(11)=	.078806650698751	UB(11)=	.394405318873305
W(12)=	.058595584986041	UB(12)=	.537483849886566
W(13)=	.048915735831746	UB(13)=	.649786289653928
W(14)=	.042967202111948	UB(14)=	.745355992499928
W(15)=	.038944135405336	UB(15)=	.829993306532575
W(16)=	.036379491801739	UB(16)=	.906764700582357
W(17)=	.034150743173670	UB(17)=	.977525219907672

Table 7 (continued)

MOMENTS

N = 16 Q(1)= -.7105E-12

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	-.3532	8.9749	15.4130	19.0471
2	-.0000	9.9461	16.0159	19.3133
3	1.5757	10.8666	16.5758	19.5386
4	3.1171	11.7385	17.0930	19.7233
5	4.4791	12.5634	17.5678	19.8679
6	5.7154	13.3426	18.0003	19.9727
7	6.8661	14.0768	18.3909	20.0381
8	7.9498	14.7668	18.7397	20.0646

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS = 12.7786

N =16 IGE =2 CYLINDERS

W(1)=	0.000000000000000	UB(1)=	-.988826464946087
W(2)=	.034150743173670	UB(2)=	-.977525219907672
W(3)=	.018189745900870	UB(3)=	-.906764700582357
W(4)=	.014693679924853	UB(4)=	-.829993306832575
W(5)=	.013586058995992	UB(5)=	-.745355992499928
W(6)=	.013586058995992	UB(6)=	-.649786289653928
W(7)=	.014693679924853	UB(7)=	-.537483849886566
W(8)=	.018189745900870	UB(8)=	-.394405318873305
W(9)=	.034150743173670	UB(9)=	-.149071198499986
W(10)=	.034150743173670	UB(10)=	.149071198499986
W(11)=	.018189745900870	UB(11)=	.394405318873305
W(12)=	.014693679924853	UB(12)=	.537483849886566
W(13)=	.013586058995992	UB(13)=	.649786289653928
W(14)=	.013586058995992	UB(14)=	.745355992499928
W(15)=	.014693679924853	UB(15)=	.829993306532575
W(16)=	.018189745900870	UB(16)=	.906764700582357
W(17)=	.034150743173670	UB(17)=	.977525219907672
W(18)=	0.000000000000000	UB(18)=	-.918934583472679
W(19)=	.018189745900870	UB(19)=	-.906764700582357
W(20)=	.009556775555629	UB(20)=	-.829993306532575
W(21)=	.007897542059982	UB(21)=	-.745355992499928
W(22)=	.007518523665789	UB(22)=	-.649786289653928
W(23)=	.007897542059982	UB(23)=	-.537483849886566
W(24)=	.009556775555629	UB(24)=	-.394405318873305
W(25)=	.018189745900870	UB(25)=	-.149071198499986
W(26)=	.018189745900870	UB(26)=	.149071198499986
W(27)=	.009556775555629	UB(27)=	.394405318873305
W(28)=	.007897542059982	UB(28)=	.537483849886566
W(29)=	.007518523665789	UB(29)=	.649786289653928
W(30)=	.007897542059982	UB(30)=	.745355992499928

Table 7 (continued)

W(31)=	.009556775555629	UB(31)=	.829993306532575
W(32)=	.018189745900870	UB(32)=	.906764700582357
W(33)=	0.000000000000000	UB(33)=	-.843274042711570
W(34)=	.014693679924853	UB(34)=	-.829993306532575
W(35)=	.007897542059982	UB(35)=	-.745355992499928
W(36)=	.006706570508185	UB(36)=	-.649786289653928
W(37)=	.006706570508183	UB(37)=	-.537483849886566
W(38)=	.007897542059982	UB(38)=	-.394405318873305
W(39)=	.014693679924853	UB(39)=	-.149071198499986
W(40)=	.014693679924853	UB(40)=	.149071198499986
W(41)=	.007897542059982	UB(41)=	.394405318873305
W(42)=	.006706570508185	UB(42)=	.537483849886566
W(43)=	.006706570508185	UB(43)=	.649786289653928
W(44)=	.007897542059982	UB(44)=	.745355992499928
W(45)=	.014693679924853	UB(45)=	.829993306532575
W(46)=	0.000000000000000	UB(46)=	-.760116950066092
W(47)=	.013586058995992	UB(47)=	-.745355992499928
W(48)=	.007518523665789	UB(48)=	-.649786289653928
W(49)=	.006706570508185	UB(49)=	-.537483849886566
W(50)=	.007518523665789	UB(50)=	-.394405318873305
W(51)=	.013586058995992	UB(51)=	-.149071198499986
W(52)=	.013586058995992	UB(52)=	.149071198499986
W(53)=	.007518523665789	UB(53)=	.394405318873305
W(54)=	.006706570508185	UB(54)=	.537483849886566
W(55)=	.007518523665789	UB(55)=	.649786289653928
W(56)=	.013586058995992	UB(56)=	.745355992499928
W(57)=	0.000000000000000	UB(57)=	-.666666666666671
W(58)=	.013586058995992	UB(58)=	-.649786289653928
W(59)=	.007897542059982	UB(59)=	-.537483849886566
W(60)=	.007897542059982	UB(60)=	-.394405318873305
W(61)=	.013586058995992	UB(61)=	-.149071198499986
W(62)=	.013586058995992	UB(62)=	.149071198499986
W(63)=	.007897542059982	UB(63)=	.394405318873305
W(64)=	.007897542059982	UB(64)=	.537483849886566
W(65)=	.013586058995992	UB(65)=	.649786289653928
W(66)=	0.000000000000000	UB(66)=	-.557773351022728
W(67)=	.014693679924853	UB(67)=	-.537483849886566
W(68)=	.009556775555629	UB(68)=	-.394405318873305
W(69)=	.014693679924853	UB(69)=	-.149071198499986
W(70)=	.014693679924853	UB(70)=	.149071198499986
W(71)=	.009556775555629	UB(71)=	.394405318873305
W(72)=	.014693679924853	UB(72)=	.537483849886566
W(73)=	0.000000000000000	UB(73)=	-.421637021355792
W(74)=	.018189745900870	UB(74)=	-.394405318873305
W(75)=	.018189745900870	UB(75)=	-.149071198499986
W(76)=	.018189745900870	UB(76)=	.149071198499986
W(77)=	.018189745900870	UB(77)=	.394405318873305
W(78)=	0.000000000000000	UB(78)=	-.210818510477925
W(79)=	.034150743173670	UB(79)=	-.149071198499986
W(80)=	.034150743173670	UB(80)=	.149071198499986

Table 7 (continued)

N =20 IGE =1 SLABS AND SPHERES

W(1)=	0.000000000000000	UB(1)=	-1.000000000000000
W(2)=	.027037345002054	UB(2)=	-.982299486257499
W(3)=	.028355452566102	UB(3)=	-.927172649945526
W(4)=	.029686530720196	UB(4)=	-.868553950490281
W(5)=	.031753818741709	UB(5)=	-.805681579172280
W(6)=	.034512680197183	UB(6)=	-.737468405508196
W(7)=	.038250093272212	UB(7)=	-.662266178532519
W(8)=	.043629782027811	UB(8)=	-.577350269189623
W(9)=	.052316254897566	UB(9)=	-.477566932940917
W(10)=	.070406017007737	UB(10)=	-.350438322025230
W(11)=	.144052025567428	UB(11)=	-.132453235706504
W(12)=	.144052025567428	UB(12)=	.132453235706504
W(13)=	.070406017007737	UB(13)=	.350438322025230
W(14)=	.052316254897566	UB(14)=	.477566932940917
W(15)=	.043629782027811	UB(15)=	.577350269189623
W(16)=	.038250093272212	UB(16)=	.662266178532519
W(17)=	.034512680197183	UB(17)=	.737468405508196
W(18)=	.031753818741709	UB(18)=	.805681579172280
W(19)=	.029686530720196	UB(19)=	.868553950490281
W(20)=	.028355452566102	UB(20)=	.927172649945526
W(21)=	.027037345002054	UB(21)=	.982299486257499

MOMENTS

N = 20 Q(1)= -.1421E-11

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	-.4288	9.3145	15.8074	19.5586
2	-.0000	10.0983	16.3021	19.7901
3	1.3583	10.8499	16.7698	19.9959
4	2.6437	11.5707	17.2107	20.1764
5	3.7851	12.2614	17.6250	20.3316
6	4.8328	12.9228	18.0128	20.4618
7	5.8171	13.5555	18.3742	20.5670
8	6.7518	14.1600	18.7094	20.6476
9	7.6437	14.7367	19.0184	20.7038
10	8.4970	15.2857	19.3014	20.7358

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS = 13.3320

Table 7 (continued)

N =24   IGE =1   SLABS AND SPHERES

W(1)=	0.000000000000000	UB(1)=	-1.000000000000000
W(2)=	.022377993115599	UB(2)=	-.985400676249824
W(3)=	.023240826176660	UB(3)=	-.940243570046871
W(4)=	.024014988139755	UB(4)=	-.892805381522397
W(5)=	.025260130757786	UB(5)=	-.842700971600383
W(6)=	.026850694116718	UB(6)=	-.789422830805581
W(7)=	.028850108118882	UB(7)=	-.732278556328101
W(8)=	.031416971700271	UB(8)=	-.670280062599833
W(9)=	.034853489567805	UB(9)=	-.601929265428844
W(10)=	.039778170417273	UB(10)=	-.524749767832798
W(11)=	.047715262911175	UB(11)=	-.434057366141213
W(12)=	.064231846597990	UB(12)=	-.318511028635301
W(13)=	.131409518380084	UB(13)=	-.120385853085769
W(14)=	.131409518380084	UB(14)=	.120385853085769
W(15)=	.064231846597990	UB(15)=	.318511028635301
W(16)=	.047715262911175	UB(16)=	.434057366141213
W(17)=	.039778170417273	UB(17)=	.524749767832798
W(18)=	.034853489567805	UB(18)=	.601929265428844
W(19)=	.031416971700271	UB(19)=	.670280062599833
W(20)=	.028850108118882	UB(20)=	.732278556328101
W(21)=	.026850694116718	UB(21)=	.789422830805581
W(22)=	.025260130757786	UB(22)=	.842700971600383
W(23)=	.024014988139755	UB(23)=	.892805381522397
W(24)=	.023240826176660	UB(24)=	.940243570046871
W(25)=	.022377993115599	UB(25)=	.985400676249824

MOMENTS

N = 24   Q(1)= -.1421E-11

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	-.4583	9.5363	16.0670	19.8942
2	-.0000	10.1933	16.4859	20.0970
3	1.1940	10.8282	16.8863	20.2823
4	2.2998	11.4417	17.2684	20.4502
5	3.2853	12.0343	17.6322	20.6008
6	4.1961	12.6065	17.9779	20.7341
7	5.0565	13.1588	18.3054	20.8504
8	5.8775	13.6913	18.6149	20.9496
9	6.6648	14.2045	18.9064	21.0319
10	7.4221	14.6984	19.1800	21.0975
11	8.1519	15.1734	19.4358	21.1464
12	8.8562	15.6295	19.6738	21.1788

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS = 13.7022

Table 7 (continued)

N =28 IGE =1 SLABS AND SPHERES

W(1)=	0.000000000000000	UB(1)=	-1.000000000000000
W(2)=	.019089083961131	UB(2)=	-.987577157479507
W(3)=	.019692950954119	UB(3)=	-.949333749479724
W(4)=	.020173737521405	UB(4)=	-.909483641319159
W(5)=	.020997480305107	UB(5)=	-.867805519545183
W(6)=	.022029006299018	UB(6)=	-.824022054121738
W(7)=	.023275663287697	UB(7)=	-.777777777777775
W(8)=	.024788751099092	UB(8)=	-.728604280477999
W(9)=	.026660780477876	UB(9)=	-.675862503366467
W(10)=	.029049244962621	UB(10)=	-.618640484758888
W(11)=	.032238140169552	UB(11)=	-.555555555555554
W(12)=	.036801966729094	UB(12)=	-.484322104837851
W(13)=	.044152724006060	UB(13)=	-.400616808384886
W(14)=	.059444810691657	UB(14)=	-.293972367896064
W(15)=	.121605659535571	UB(15)=	-.111111111111111
W(16)=	.121605659535571	UB(16)=	.111111111111111
W(17)=	.059444810691657	UB(17)=	.293972367896064
W(18)=	.044152724006060	UB(18)=	.400616808384886
W(19)=	.036801966729094	UB(19)=	.484322104837851
W(20)=	.032238140169552	UB(20)=	.555555555555554
W(21)=	.029049244962621	UB(21)=	.618640484758888
W(22)=	.026660780477876	UB(22)=	.675862503366467
W(23)=	.024788751099092	UB(23)=	.728604280477999
W(24)=	.023275663287697	UB(24)=	.777777777777775
W(25)=	.022029006299018	UB(25)=	.824022054121738
W(26)=	.020997480305107	UB(26)=	.867805519545183
W(27)=	.020173737521405	UB(27)=	.909483641319159
W(28)=	.019692950954119	UB(28)=	.949333749479724
W(29)=	.019089083961131	UB(29)=	.987577157479507

MOMENTS

N = 28 Q(1)= -.2132E-11

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	-.4669	9.6916	16.2505	20.1312
2	-.0000	10.2573	16.6136	20.3110
3	1.0661	10.8069	16.9633	20.4780
4	2.0382	11.3407	17.2996	20.6324
5	2.9072	11.8591	17.6226	20.7742
6	3.7140	12.3625	17.9323	20.9034
7	4.4790	12.8511	18.2289	21.0201
8	5.2113	13.3251	18.5122	21.1244
9	5.9159	13.7849	18.7825	21.2163
10	6.5959	14.2305	19.0397	21.2959
11	7.2538	14.6620	19.2838	21.3633
12	7.8912	15.0797	19.5150	21.4185
13	8.5093	15.4836	19.7333	21.4617
14	9.1091	15.8738	19.9387	21.4929

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS = 13.9668

Table 7 (continued)

N =32   IGE =1   SLABS AND SPHERES

W(1)=	0.000000000000000	UB(1)=	-1.000000000000000
W(2)=	.016643326757938	UB(2)=	-.989188871579085
W(3)=	.017086447816649	UB(3)=	-.956022224963231
W(4)=	.017396546590761	UB(4)=	-.921662826412884
W(5)=	.017976469903380	UB(5)=	-.885971916349007
W(6)=	.018697204851399	UB(6)=	-.848781542875948
W(7)=	.019548370100563	UB(7)=	-.809885163769909
W(8)=	.020548030821511	UB(8)=	-.769023958965100
W(9)=	.021732162935595	UB(9)=	-.725866186311293
W(10)=	.023157934689386	UB(10)=	-.679974699086550
W(11)=	.024915526382391	UB(11)=	-.630753091443545
W(12)=	.027153927401173	UB(12)=	-.577350269189623
W(13)=	.030139622538597	UB(13)=	-.518475847365210
W(14)=	.034410426410850	UB(14)=	-.451996764666314
W(15)=	.041287309670913	UB(15)=	-.373879250552981
W(16)=	.055591822573321	UB(16)=	-.274351630584366
W(17)=	.113714870555570	UB(17)=	-.103695169473042
W(18)=	.113714870555570	UB(18)=	.103695169473042
W(19)=	.055591822573321	UB(19)=	.274351630584366
W(20)=	.041287309670913	UB(20)=	.373879250552981
W(21)=	.034410426410850	UB(21)=	.451996764666314
W(22)=	.030139622538597	UB(22)=	.518475847365210
W(23)=	.027153927401173	UB(23)=	.577350269189623
W(24)=	.024915526382391	UB(24)=	.630753091443545
W(25)=	.023157934689386	UB(25)=	.679974699086550
W(26)=	.021732162935595	UB(26)=	.725866186311293
W(27)=	.020548030821511	UB(27)=	.769023958965100
W(28)=	.019548370100563	UB(28)=	.809885163769909
W(29)=	.018697204851399	UB(29)=	.848781542875948
W(30)=	.017976469903380	UB(30)=	.885971916349007
W(31)=	.017396546590761	UB(31)=	.921662826412884
W(32)=	.017086447816649	UB(32)=	.956022224963231
W(33)=	.016643326757938	UB(33)=	.989188871579085

MOMENTS

N =   32   Q(1)= -.2132E-11

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	-.4653	9.8062	16.3869	20.3075
2	-.0000	10.3030	16.7073	20.4686
3	.9638	10.7874	17.0175	20.6201
4	1.8323	11.2597	17.3175	20.7619
5	2.6105	11.7203	17.6074	20.8941
6	3.3356	12.1692	17.8871	21.0168
7	4.0248	12.6068	18.1568	21.1300
8	4.6862	13.0333	18.4165	21.2337
9	5.3240	13.4486	18.6661	21.3279
10	5.9413	13.8531	18.9059	21.4127
11	6.5399	14.2467	19.1356	21.4882
12	7.1215	14.6297	19.3555	21.5545

Table 7 (continued)

13	7.6871	15.0020	19.5655	21.6114
14	8.2377	15.3639	19.7657	21.6592
15	8.7740	15.7152	19.9560	21.6978
16	9.2967	16.0562	20.1366	21.7273

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS = 14.1652

N =40 IGE =1 SLABS AND SPHERES

W(1)=	0.000000000000000	UB(1)=	-1.000000000000000
W(2)=	.013248768917978	UB(2)=	-.991416150214416
W(3)=	.013511872300549	UB(3)=	-.965206678190697
W(4)=	.013645718156071	UB(4)=	-.938265357104196
W(5)=	.013970258687021	UB(5)=	-.910527225875111
W(6)=	.014375504662528	UB(6)=	-.881917103688192
W(7)=	.014843178512361	UB(7)=	-.852347186594592
W(8)=	.015373254381062	UB(8)=	-.821713864562152
W(9)=	.015972929940773	UB(9)=	-.789893425679448
W(10)=	.016654518271053	UB(10)=	-.756734131455057
W(11)=	.017435828188347	UB(11)=	-.722057837965568
W(12)=	.018341971065111	UB(12)=	-.685627792672864
W(13)=	.019408827883381	UB(13)=	-.647150229929430
W(14)=	.020689293919767	UB(14)=	-.606235406027530
W(15)=	.022264949977925	UB(15)=	-.562351594857979
W(16)=	.024269565800262	UB(16)=	-.514730097432937
W(17)=	.026941775687476	UB(17)=	-.462250163521022
W(18)=	.030762751987602	UB(18)=	-.402980349884656
W(19)=	.036914019948345	UB(19)=	-.333333333333332
W(20)=	.049708142380843	UB(20)=	-.244599795235113
W(21)=	.101666869331541	UB(21)=	-.092450032704205
W(22)=	.101666869331541	UB(22)=	.092450032704205
W(23)=	.049708142380843	UB(23)=	.244599795235113
W(24)=	.036914019948345	UB(24)=	.333333333333332
W(25)=	.030762751987602	UB(25)=	.402980349884656
W(26)=	.026941775687476	UB(26)=	.462250163521022
W(27)=	.024269565800262	UB(27)=	.514730097432937
W(28)=	.022264949977925	UB(28)=	.562351594857979
W(29)=	.020689293919767	UB(29)=	.606235406027530
W(30)=	.019408827883381	UB(30)=	.647150229929430
W(31)=	.018341971065111	UB(31)=	.685627792672864
W(32)=	.017435828188347	UB(32)=	.722057837965568
W(33)=	.016654518271053	UB(33)=	.756734131455057
W(34)=	.015972929940773	UB(34)=	.789893425679448
W(35)=	.015373254381062	UB(35)=	.821713864562152
W(36)=	.014843178512361	UB(36)=	.852347186594592
W(37)=	.014375504662528	UB(37)=	.881917103688192
W(38)=	.013970258687021	UB(38)=	.910527225875111
W(39)=	.013645718156071	UB(39)=	.938265357104196
W(40)=	.013511872300549	UB(40)=	.965206678190697
W(41)=	.013248768917978	UB(41)=	.991416150214416

Table 7 (continued)

MOMENTS

N = 40 Q(1)= -.3553E-11

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	-.4492	9.9636	16.5762	20.5520
2	-.0000	10.3631	16.8355	20.6851
3	.8102	10.7547	17.0883	20.8121
4	1.5280	11.1386	17.3347	20.9331
5	2.1736	11.5149	17.5746	21.0479
6	2.7779	11.8837	17.8080	21.1566
7	3.3545	12.2452	18.0351	21.2593
8	3.9096	12.5994	18.2558	21.3560
9	4.4468	12.9465	18.4701	21.4466
10	4.9684	13.2864	18.6781	21.5313
11	5.4760	13.6194	18.8798	21.6100
12	5.9710	13.9455	19.0751	21.6828
13	6.4542	14.2646	19.2642	21.7496
14	6.9264	14.5770	19.4470	21.8105
15	7.3881	14.8825	19.6235	21.8656
16	7.8399	15.1814	19.7937	21.9148
17	8.2823	15.4736	19.9578	21.9582
18	8.7156	15.7591	20.1156	21.9958
19	9.1400	16.0380	20.2672	22.0276
20	9.5559	16.3104	20.4127	22.0536

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS = 14.4427

N =48 IGE =1 SLABS AND SPHERES

W(1)=	0.000000000000000	UB(1)=	-1.000000000000000
W(2)=	.011004500018045	UB(2)=	-.992882471814699
W(3)=	.011175073723283	UB(3)=	-.971216974392181
W(4)=	.011227862781370	UB(4)=	-.949057016126083
W(5)=	.011429416333522	UB(5)=	-.926367113173168
W(6)=	.011686132073598	UB(6)=	-.903107323012843
W(7)=	.011980019417486	UB(7)=	-.879232418300223
W(8)=	.012307366916966	UB(8)=	-.854690852823815
W(9)=	.012669207657044	UB(9)=	-.829423451809351
W(10)=	.013069005521269	UB(10)=	-.803361731053940
W(11)=	.013512086032125	UB(11)=	-.776425707606734
W(12)=	.014005691553957	UB(12)=	-.748521000335501
W(13)=	.014559396718087	UB(13)=	-.719534916771863
W(14)=	.015185873794486	UB(14)=	-.689331056143949
W(15)=	.015902133357104	UB(15)=	-.657741676857391
W(16)=	.016731525690443	UB(16)=	-.624556580291816
W(17)=	.017707060623145	UB(17)=	-.589506344746560
W(18)=	.018877182513209	UB(18)=	-.552235945051649
W(19)=	.020316463516794	UB(19)=	-.512261015028205
W(20)=	.022147078324144	UB(20)=	-.468890345420299
W(21)=	.024586900004138	UB(21)=	-.421075960533258
W(22)=	.028075179066400	UB(22)=	-.367085511903758
W(23)=	.033690465928980	UB(23)=	-.303642193313582
W(24)=	.045369627928145	UB(24)=	-.222812454927729
W(25)=	.092784750506257	UB(25)=	-.084215192106652
W(26)=	.092784750506257	UB(26)=	.084215192106652
W(27)=	.045369627928145	UB(27)=	.222812454927729

Table 7 (continued)

W(28)=	.033690465928980	UB(28)=	.303642193313582
W(29)=	.028075179066400	UB(29)=	.367085511903758
W(30)=	.024586900004138	UB(30)=	.421075960533258
W(31)=	.022147078324144	UB(31)=	.468890345420299
W(32)=	.020316463516794	UB(32)=	.512261015028205
W(33)=	.018877182513209	UB(33)=	.552235945051649
W(34)=	.017707060623145	UB(34)=	.589504344746560
W(35)=	.016731525690443	UB(35)=	.624556580291816
W(36)=	.015902133357104	UB(36)=	.657741676857391
W(37)=	.015185873794486	UB(37)=	.689331056143949
W(38)=	.014559396718087	UB(38)=	.719534914771863
W(39)=	.014005691553957	UB(39)=	.748521000335501
W(40)=	.013512086032125	UB(40)=	.776425707606734
W(41)=	.013069005521269	UB(41)=	.803361731053940
W(42)=	.012669207657044	UB(42)=	.829423451809351
W(43)=	.012307366916966	UB(43)=	.854690852923815
W(44)=	.011980019417486	UB(44)=	.879232418300223
W(45)=	.011686132073598	UB(45)=	.903107323012843
W(46)=	.011429416333522	UB(46)=	.926367113173168
W(47)=	.011227862781370	UB(47)=	.949057016126083
W(48)=	.011175073723283	UB(48)=	.971216974392181
W(49)=	.011004500018045	UB(49)=	.992882471814699

MOMENTS

N = 48 Q(1)= -.4263E-11

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	-.4275	10.0663	16.7012	20.7135
2	-.0000	10.4005	16.9190	20.8268
3	.7002	10.7293	17.1322	20.9358
4	1.3133	11.0526	17.3410	21.0407
5	1.8662	11.3706	17.5453	21.1413
6	2.3854	11.6834	17.7452	21.2377
7	2.8820	11.9910	17.9407	21.3300
8	3.3612	12.2936	18.1317	21.4181
9	3.8261	12.5911	18.3183	21.5020
10	4.2785	12.8836	18.5006	21.5817
11	4.7198	13.1712	18.6784	21.6574
12	5.1511	13.4540	18.8519	21.7289
13	5.5731	13.7319	19.0210	21.7962
14	5.9865	14.0050	19.1857	21.8595
15	6.3918	14.2733	19.3462	21.9187
16	6.7893	14.5369	19.5022	21.9739
17	7.1795	14.7959	19.6540	22.0249
18	7.5626	15.0501	19.8014	22.0720
19	7.9390	15.2997	19.9446	22.1150
20	8.3088	15.5447	20.0834	22.1539
21	8.6723	15.7851	20.2180	22.1889
22	9.0297	16.0210	20.3483	22.2199
23	9.3810	16.2523	20.4743	22.2469
24	9.7265	16.4790	20.5960	22.2700

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS = 14.6273

Table 7 (continued)

N =56 IGE =1 SLABS AND SPHERES

W(1)=	0.000000000000000	UB(1)=	-1.000000000000000
W(2)=	.009410493374558	UB(2)=	-.993920916310135
W(3)=	.009527733116603	UB(3)=	-.975456381144305
W(4)=	.009538827066233	UB(4)=	-.956635518445509
W(5)=	.009672557286623	UB(5)=	-.937436866561086
W(6)=	.009848000590556	UB(6)=	-.917836718825431
W(7)=	.010048743524827	UB(7)=	-.897808780342782
W(8)=	.010270333479242	UB(8)=	-.877323754207627
W(9)=	.010512039845773	UB(9)=	-.856348838577670
W(10)=	.010774762258558	UB(10)=	-.834847109936717
W(11)=	.011060382690950	UB(11)=	-.812776759390946
W(12)=	.011371579376893	UB(12)=	-.790090136783402
W(13)=	.011711834736597	UB(13)=	-.766732539989523
W(14)=	.012085558966283	UB(14)=	-.742640661097372
W(15)=	.012498316256336	UB(15)=	-.717740562565268
W(16)=	.012957170782739	UB(16)=	-.691944096938251
W(17)=	.013471202656450	UB(17)=	-.665149789464177
W(18)=	.014052279882001	UB(18)=	-.637228849049226
W(19)=	.014716239847377	UB(19)=	-.608027112633117
W(20)=	.015484759360071	UB(20)=	-.577350269189623
W(21)=	.016388439531903	UB(21)=	-.544949260913064
W(22)=	.017472158345081	UB(22)=	-.510495896757316
W(23)=	.018804972955694	UB(23)=	-.473542420742241
W(24)=	.020500004530006	UB(24)=	-.433449867780331
W(25)=	.022758965151359	UB(25)=	-.389249472080760
W(26)=	.025988511907099	UB(26)=	-.339339822525318
W(27)=	.031187167731323	UB(27)=	-.280691786106894
W(28)=	.041999826272762	UB(28)=	-.205971460217774
W(29)=	.085887138476101	UB(29)=	-.077849894416152
W(30)=	.085887138476101	UB(30)=	.077849894416152
W(31)=	.041999826272762	UB(31)=	.205971460217774
W(32)=	.031187167731323	UB(32)=	.280691786106894
W(33)=	.025988511907099	UB(33)=	.339339822525318
W(34)=	.022758965151359	UB(34)=	.389249472080760
W(35)=	.020500004530006	UB(35)=	.433449867780331
W(36)=	.018804972955694	UB(36)=	.473542420742241
W(37)=	.017472158345081	UB(37)=	.510495896757316
W(38)=	.016388439531903	UB(38)=	.544949260913064
W(39)=	.015484759360071	UB(39)=	.577350269189623
W(40)=	.014716239847377	UB(40)=	.608027112633117
W(41)=	.014052279882001	UB(41)=	.637228849049226
W(42)=	.013471202656450	UB(42)=	.665149789464177
W(43)=	.012957170782739	UB(43)=	.691944096938251
W(44)=	.012498316256336	UB(44)=	.717740562565268
W(45)=	.012085558966283	UB(45)=	.742640661097372
W(46)=	.011711834736597	UB(46)=	.766732539989523
W(47)=	.011371579376893	UB(47)=	.790090136783402
W(48)=	.011060382690950	UB(48)=	.812776759390946
W(49)=	.010774762258558	UB(49)=	.834847109936717
W(50)=	.010512039845773	UB(50)=	.856348838577670
W(51)=	.010270333479242	UB(51)=	.877323754207627
W(52)=	.010048743524827	UB(52)=	.897808780342782
W(53)=	.009848000590556	UB(53)=	.917836718825431
W(54)=	.009672557286623	UB(54)=	.937436866561086
W(55)=	.009538827066233	UB(55)=	.956635518445509
W(56)=	.009527733116603	UB(56)=	.975456381144305
W(57)=	.009410493374558	UB(57)=	.993920916310135

Table 7 (continued)

MOMENTS

N = 56 Q(1)= -.4263E-11

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	-.4054	10.1387	16.7899	20.8282
2	-.0000	10.4259	16.9776	20.9267
3	.6174	10.7092	17.1619	21.0221
4	1.1533	10.9884	17.3430	21.1144
5	1.6375	11.2638	17.5208	21.2037
6	2.0933	11.5353	17.6954	21.2898
7	2.5300	11.8029	17.8667	21.3730
8	2.9522	12.0668	18.0348	21.4530
9	3.3625	12.3270	18.1996	21.5300
10	3.7625	12.5834	18.3613	21.6040
11	4.1534	12.8362	18.5197	21.6749
12	4.5360	13.0854	18.6748	21.7428
13	4.9110	13.3310	18.8268	21.8077
14	5.2789	13.5730	18.9756	21.8695
15	5.6402	13.8115	19.1212	21.9284
16	5.9952	14.0464	19.2637	21.9842
17	6.3443	14.2779	19.4029	22.0371
18	6.6876	14.5059	19.5390	22.0870
19	7.0255	14.7305	19.6719	22.1339
20	7.3581	14.9516	19.8016	22.1779
21	7.6856	15.1693	19.9282	22.2189
22	8.0082	15.3836	20.0517	22.2569
23	8.3260	15.5945	20.1720	22.2921
24	8.6392	15.8021	20.2891	22.3243
25	8.9478	16.0063	20.4032	22.3535
26	9.2519	16.2072	20.5141	22.3799
27	9.5517	16.4048	20.6219	22.4034
28	9.8473	16.5990	20.7266	22.4239

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS = 14.7589

Table 7 (continued)

N =64 IGE =1 SLABS AND SPHERES

W(1)=	0.000000000000000	UB(1)=	-1.000000000000000
W(2)=	.008219876939752	UB(2)=	-.994094922796870
W(3)=	.008303837861379	UB(3)=	-.978607151860206
W(4)=	.008291952400885	UB(4)=	-.962250448649371
W(5)=	.008384734563119	UB(5)=	-.945610857689296
W(6)=	.008511097318607	UB(6)=	-.928673173099046
W(7)=	.008656256205577	UB(7)=	-.911420775870191
W(8)=	.008815786164431	UB(8)=	-.893835442876256
W(9)=	.008988398854562	UB(9)=	-.875897121353734
W(10)=	.009174084194021	UB(10)=	-.857583660904130
W(11)=	.009373503238252	UB(11)=	-.838870492807857
W(12)=	.009587769435679	UB(12)=	-.819730243407953
W(13)=	.009818378314971	UB(13)=	-.800132264198634
W(14)=	.010067208880779	UB(14)=	-.780042055574956
W(15)=	.010336567762431	UB(15)=	-.759420553263848
W(16)=	.010629269509619	UB(16)=	-.738223235189423
W(17)=	.010948757908892	UB(17)=	-.716398990247409
W(18)=	.011299275245663	UB(18)=	-.693888666498707
W(19)=	.011686102764158	UB(19)=	-.670623180135795
W(20)=	.012115900163404	UB(20)=	-.646521011251309
W(21)=	.012597197141915	UB(21)=	-.621484823823867
W(22)=	.013141120263043	UB(22)=	-.595396804238444
W(23)=	.013762501691879	UB(23)=	-.568112068830896
W(24)=	.014481631434089	UB(24)=	-.539449062475121
W(25)=	.015327143496272	UB(25)=	-.509175077217314
W(26)=	.016341022606168	UB(26)=	-.476983466708464
W(27)=	.017587869672862	UB(27)=	-.442455868722739
W(28)=	.019173499951486	UB(28)=	-.404995264195968
W(29)=	.021286594515999	UB(29)=	-.363696483726653
W(30)=	.024307537831514	UB(30)=	-.317063243737113
W(31)=	.029170350507264	UB(31)=	-.262265264156481
W(32)=	.039284579404757	UB(32)=	-.192450089729874
W(33)=	.080330191756567	UB(33)=	-.072730296745331
W(34)=	.080330191756567	UB(34)=	.072730296745331
W(35)=	.039284579404757	UB(35)=	.192450089729874
W(36)=	.029170350507264	UB(36)=	.262265264156481
W(37)=	.024307537831514	UB(37)=	.317063243737113
W(38)=	.021286594515999	UB(38)=	.363696483726653
W(39)=	.019173499951486	UB(39)=	.404995264195968
W(40)=	.017587869672862	UB(40)=	.442455868722739
W(41)=	.016341022606168	UB(41)=	.476983466708464
W(42)=	.015327143496272	UB(42)=	.509175077217314
W(43)=	.014481631434089	UB(43)=	.539449062475121
W(44)=	.013762501691879	UB(44)=	.568112068830896
W(45)=	.013141120263043	UB(45)=	.595396804238444
W(46)=	.012597197141915	UB(46)=	.621484823823867
W(47)=	.012115900163404	UB(47)=	.646521011251309
W(48)=	.011686102764158	UB(48)=	.670623180135795
W(49)=	.011299275245663	UB(49)=	.693888666498707
W(50)=	.010948757908892	UB(50)=	.716398990247409
W(51)=	.010629269509619	UB(51)=	.738223235189423
W(52)=	.010336567762431	UB(52)=	.759420553263848
W(53)=	.010067208880779	UB(53)=	.780042055574956
W(54)=	.009818378314971	UB(54)=	.800132264198634
W(55)=	.009587769435679	UB(55)=	.819730243407953

Table 7 (continued)

W(56)=	.009373503238252	UB(56)=	.838870492807857
W(57)=	.009174084194021	UB(57)=	.857583660904130
W(58)=	.008988398854562	UB(58)=	.875897121353734
W(59)=	.008815786164431	UB(59)=	.893835442876256
W(60)=	.008656256205577	UB(60)=	.911420775870191
W(61)=	.008511097318607	UB(61)=	.928673173099046
W(62)=	.008384734563119	UB(62)=	.945610857689296
W(63)=	.008291952400885	UB(63)=	.962250448649371
W(64)=	.008303839861379	UB(64)=	.978607151860206
W(65)=	.008219876939752	UB(65)=	.994694922786870

MOMENTS

N = 64 Q(1)= -.5684E-11

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	-.3846	10.1923	16.8561	20.9138
2	-.0000	10.4442	17.0210	21.0009
3	.5527	10.6930	17.1833	21.0857
4	1.0291	10.9388	17.3431	21.1680
5	1.4603	11.1816	17.5005	21.2481
6	1.8669	11.4213	17.6553	21.3258
7	2.2571	11.6582	17.8077	21.4011
8	2.6349	11.8921	17.9576	21.4741
9	3.0025	12.1232	18.1050	21.5448
10	3.3614	12.3513	18.2500	21.6132
11	3.7125	12.5767	18.3925	21.6792
12	4.0566	12.7993	18.5325	21.7430
13	4.3944	13.0190	18.6701	21.8044
14	4.7262	13.2361	18.8053	21.8635
15	5.0524	13.4504	18.9380	21.9203
16	5.3733	13.6619	19.0682	21.9748
17	5.6892	13.8708	19.1961	22.0270
18	6.0004	14.0769	19.3215	22.0769
19	6.3070	14.2804	19.4445	22.1246
20	6.6092	14.4813	19.5650	22.1699
21	6.9072	14.6795	19.6831	22.2130
22	7.2010	14.8751	19.7989	22.2539
23	7.4910	15.0680	19.9122	22.2925
24	7.7770	15.2584	20.0231	22.3288
25	8.0593	15.4462	20.1316	22.3629
26	8.3379	15.6313	20.2377	22.3947
27	8.6130	15.8140	20.3415	22.4243
28	8.8846	15.9940	20.4428	22.4517
29	9.1527	16.1715	20.5417	22.4769
30	9.4175	16.3464	20.6383	22.4998
31	9.6790	16.5189	20.7325	22.5206
32	9.9372	16.6887	20.8244	22.5391

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS = 14.8574

TABLE 8

AREA METHOD (AM) WEIGHTS AND DIRECTIONS

N = 2 IGE =1 SLABS AND SPHERES

W(1)=	0.000000000000000	UB(1)=	-1.000000000000000
W(2)=	.500000000000000	UB(2)=	-.577350269189626
W(3)=	.500000000000000	UB(3)=	.577350269189626

MOMENTS

N = 2 Q(1)= 0.

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	15.4701	0.0000	-23.0200	-44.4444

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS = 10.5869

N = 2 IGE =2 CYLINDERS

W(1)=	0.000000000000000	UB(1)=	-.816496580927726
W(2)=	.500000000000000	UB(2)=	-.577350269189626
W(3)=	.500000000000000	UB(3)=	.577350269189626

N = 4 IGE =1 SLABS AND SPHERES

W(1)=	0.000000000000000	UB(1)=	-1.000000000000000
W(2)=	.166666666666666	UB(2)=	-.873686442609706
W(3)=	.333333333333332	UB(3)=	-.343999999999999
W(4)=	.333333333333332	UB(4)=	.343999999999999
W(5)=	.166666666666666	UB(5)=	.873686442609706

MOMENTS

N = 4 Q(1)= 0.

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	4.1124	-.2234	3.7410	3.9274
2	0.0000	1.7794	4.5525	1.9688

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS = 2.2561

N = 4 IGE =2 CYLINDERS

W(1)=	0.000000000000000	UB(1)=	-.938969648071755
W(2)=	.166666666666666	UB(2)=	-.873686442609706
W(3)=	.166666666666666	UB(3)=	-.343999999999999
W(4)=	.166666666666666	UB(4)=	.343999999999999
W(5)=	.166666666666666	UB(5)=	.873686442609706
W(6)=	0.000000000000000	UB(6)=	-.486489465456343
W(7)=	.166666666666666	UB(7)=	-.343999999999999
W(8)=	.166666666666666	UB(8)=	.343999999999999

TABLE 8 (continued)

N = 6   IGE =1   SLABS AND SPHERES

W(1)=	0.000000000000000	UB(1)=	-1.000000000000000
W(2)=	.100451046290544	UB(2)=	-.917837003656793
W(3)=	.132431240752243	UB(3)=	-.678679741351118
W(4)=	.267117712957210	UB(4)=	-.280691320420065
W(5)=	.267117712957210	UB(5)=	.280691320420065
W(6)=	.132431240752243	UB(6)=	.678679741351118
W(7)=	.100451046290544	UB(7)=	.917837003656793

MOMENTS

N = 6   Q(1)= 0.

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	2.8215	1.0425	2.3044	-.1946
2	-.0000	1.9576	1.8131	-1.6443
3	-.0196	2.3775	.9666	-3.3617

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS = 1.4233

N = 6   IGE =2   CYLINDERS

W(1)=	0.000000000000000	UB(1)=	-.959798094726615
W(2)=	.100451046290544	UB(2)=	-.917837003656793
W(3)=	.066215620376122	UB(3)=	-.678679741351118
W(4)=	.100451046290544	UB(4)=	-.280691320420065
W(5)=	.100451046290544	UB(5)=	.280691320420065
W(6)=	.066215620376122	UB(6)=	.678679741351118
W(7)=	.100451046290544	UB(7)=	.917837003656793
W(8)=	0.000000000000000	UB(8)=	-.734434346064763
W(9)=	.066215620376122	UB(9)=	-.678679741351118
W(10)=	.066215620376122	UB(10)=	-.280691320420065
W(11)=	.066215620376122	UB(11)=	.280691320420065
W(12)=	.066215620376122	UB(12)=	.678679741351118
W(13)=	0.000000000000000	UB(13)=	-.396957472178473
W(14)=	.100451046290544	UB(14)=	-.280691320420065
W(15)=	.100451046290544	UB(15)=	.280691320420065

N = 8   IGE =1   SLABS AND SPHERES

W(1)=	0.000000000000000	UB(1)=	-1.000000000000000
W(2)=	.072242022223608	UB(2)=	-.939121295338374
W(3)=	.085280409813437	UB(3)=	-.779513354821891
W(4)=	.112713113702299	UB(4)=	-.577350269189623
W(5)=	.229764454260654	UB(5)=	-.242951839509359
W(6)=	.229764454260654	UB(6)=	.242951839509359
W(7)=	.112713113702299	UB(7)=	.577350269189623
W(8)=	.085280409813437	UB(8)=	.779513354821891
W(9)=	.072242022223608	UB(9)=	.939121295338374

MOMENTS

N = 8 Q(1)= 0.

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	2.0872	1.6900	1.8284	-.9822
2	-.0000	2.0794	1.3694	-2.0673
3	.1726	2.2048	.7434	-3.2898
4	1.0043	2.1118	-.0428	-4.6410

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS = 1.5479

N = 8 IGE =2 CYLINDERS

W(1)=	0.000000000000000	UB(1)=	-.970038351653692
W(2)=	.072242022223608	UB(2)=	-.939121295338374
W(3)=	.042640204906719	UB(3)=	-.779513354821891
W(4)=	.042640204906719	UB(4)=	-.577350269189623
W(5)=	.072242022223608	UB(5)=	-.242951839509359
W(6)=	.072242022223608	UB(6)=	.242951839509359
W(7)=	.042640204906719	UB(7)=	.577350269189623
W(8)=	.042640204906719	UB(8)=	.779513354821891
W(9)=	.072242022223608	UB(9)=	.939121295338374
W(10)=	0.000000000000000	UB(10)=	-.816496580927726
W(11)=	.042640204906719	UB(11)=	-.779513354821891
W(12)=	.027432703888862	UB(12)=	-.577350269189623
W(13)=	.042640204906719	UB(13)=	-.242951839509359
W(14)=	.042640204906719	UB(14)=	.242951839509359
W(15)=	.027432703888862	UB(15)=	.577350269189623
W(16)=	.042640204906719	UB(16)=	.779513354821891
W(17)=	0.000000000000000	UB(17)=	-.626385607796283
W(18)=	.042640204906719	UB(18)=	-.577350269189623
W(19)=	.042640204906719	UB(19)=	-.242951839509359
W(20)=	.042640204906719	UB(20)=	.242951839509359
W(21)=	.042640204906719	UB(21)=	.577350269189623
W(22)=	0.000000000000000	UB(22)=	-.343585786437639
W(23)=	.072242022223608	UB(23)=	-.242951839509359
W(24)=	.072242022223608	UB(24)=	.242951839509359

N =10 IGE =1 SLABS AND SPHERES

W(1)=	0.000000000000000	UB(1)=	-1.000000000000000
W(2)=	.056462466014014	UB(2)=	-.951656864964573
W(3)=	.063585277886588	UB(3)=	-.831283190553240
W(4)=	.074919355157779	UB(4)=	-.690226554950293
W(5)=	.100222258635287	UB(5)=	-.511657748324719
W(6)=	.204810642306331	UB(6)=	-.217197158551618
W(7)=	.204810642306331	UB(7)=	.217197158551618
W(8)=	.100222258635287	UB(8)=	.511657748324719
W(9)=	.074919355157779	UB(9)=	.690226554950293
W(10)=	.063585277886588	UB(10)=	.831283190553240
W(11)=	.056462466014014	UB(11)=	.951656864964573

Table 8 (continued)

MOMENTS

N = 10 Q(1)= -.7105E-12

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	1.6262	1.9820	1.7567	-1.1941
2	-.0000	2.1955	1.3657	-2.0629
3	.2788	2.2666	.8712	-3.0150
4	1.0032	2.2120	.2770	-4.0460
5	1.5956	2.0402	-.4127	-5.1515

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS = 1.6835

N =10 IGE =2 CYLINDERS

W(1)=	0.000000000000000	UR(1)=	-.976127755120764
W(2)=	.056462466014014	UR(2)=	-.951656864964573
W(3)=	.031792638943294	UR(3)=	-.831283190553240
W(4)=	.028300432391715	UR(4)=	-.690226554950293
W(5)=	.031792638943294	UR(5)=	-.511657748324719
W(6)=	.056462466014014	UR(6)=	-.217197158551618
W(7)=	.056462466014014	UR(7)=	.217197158551618
W(8)=	.031792638943294	UR(8)=	.511657748324719
W(9)=	.028300432391715	UR(9)=	.690226554950293
W(10)=	.031792638943294	UR(10)=	.831283190553240
W(11)=	.056462466014014	UR(11)=	.951656864964573
W(12)=	0.000000000000000	UR(12)=	-.859189355485316
W(13)=	.031792638943294	UR(13)=	-.831283190553240
W(14)=	.018318490374349	UR(14)=	-.690226554950293
W(15)=	.018318490374349	UR(15)=	-.511657748324719
W(16)=	.031792638943294	UR(16)=	-.217197158551618
W(17)=	.031792638943294	UR(17)=	.217197158551618
W(18)=	.018318490374349	UR(18)=	.511657748324719
W(19)=	.018318490374349	UR(19)=	.690226554950293
W(20)=	.031792638943294	UR(20)=	.831283190553240
W(21)=	0.000000000000000	UR(21)=	-.723593326974100
W(22)=	.028300432391715	UR(22)=	-.690226554950293
W(23)=	.018318490374349	UR(23)=	-.511657748324719
W(24)=	.028300432391715	UR(24)=	-.217197158551618
W(25)=	.028300432391715	UR(25)=	.217197158551618
W(26)=	.018318490374349	UR(26)=	.511657748324719
W(27)=	.028300432391715	UR(27)=	.690226554950293
W(28)=	0.000000000000000	UR(28)=	-.555849131602830
W(29)=	.031792638943294	UR(29)=	-.511657748324719
W(30)=	.031792638943294	UR(30)=	-.217197158551618
W(31)=	.031792638943294	UR(31)=	.217197158551618
W(32)=	.031792638943294	UR(32)=	.511657748324719
W(33)=	0.000000000000000	UR(33)=	-.307163167332613
W(34)=	.056462466014014	UR(34)=	-.217197158551618
W(35)=	.056462466014014	UR(35)=	.217197158551618

Table 8 (continued)

N =12   IGE =1   SLABS AND SPHERES

W(1)=	0.000000000000000	UB(1)=	-1.000000000000000
W(2)=	.046357556555418	UB(2)=	-.959919133766732
W(3)=	.050838290719928	UB(3)=	-.863140382765184
W(4)=	.057003591715130	UB(4)=	-.754041044869965
W(5)=	.067974072877962	UB(5)=	-.626214399656497
W(6)=	.091233466775218	UB(6)=	-.464447038235473
W(7)=	.186593021356344	UB(7)=	-.198185842870421
W(8)=	.186593021356344	UB(8)=	.198185842870421
W(9)=	.091233466775218	UB(9)=	.464447038235473
W(10)=	.067974072877962	UB(10)=	.626214399656497
W(11)=	.057003591715130	UB(11)=	.754041044869965
W(12)=	.050838290719928	UB(12)=	.863140382765184
W(13)=	.046357556555418	UB(13)=	.959919133766732

MOMENTS

N = 12   Q(1)= -.7105E-12

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	1.3131	2.1681	1.7864	-1.2449
2	-.0000	2.3192	1.4490	-1.9684
3	.3357	2.3776	1.0413	-2.7478
4	.9919	2.3496	.5657	-3.5809
5	1.5301	2.2396	.0245	-4.4649
6	1.9124	2.0508	-.5798	-5.3975

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS = 1.7776

N =12   IGE =2   CYLINDERS

W(1)=	0.000000000000000	UB(1)=	-.980164461550071
W(2)=	.046357556555418	UB(2)=	-.959919133766732
W(3)=	.025419145359964	UB(3)=	-.863140382765184
W(4)=	.021519808762790	UB(4)=	-.754041044869965
W(5)=	.021519808762790	UB(5)=	-.626214399656497
W(6)=	.025419145359964	UB(6)=	-.464447038235473
W(7)=	.046357556555418	UB(7)=	-.198185842870421
W(8)=	.046357556555418	UB(8)=	.198185842870421
W(9)=	.025419145359964	UB(9)=	.464447038235473
W(10)=	.021519808762790	UB(10)=	.626214399656497
W(11)=	.021519808762790	UB(11)=	.754041044869965
W(12)=	.025419145359964	UB(12)=	.863140382765184
W(13)=	.046357556555418	UB(13)=	.959919133766732
W(14)=	0.000000000000000	UB(14)=	-.885600896947544
W(15)=	.025419145359964	UB(15)=	-.863140382765184
W(16)=	.013963974189550	UB(16)=	-.754041044869965
W(17)=	.012467227676191	UB(17)=	-.626214399656497
W(18)=	.013963974189550	UB(18)=	-.464447038235473
W(19)=	.025419145359964	UB(19)=	-.198185842870421
W(20)=	.025419145359964	UB(20)=	.198185842870421

Table 8 (continued)

W(21)=	.013963974189550	UB(21)=	.464447038235473
W(22)=	.012467227676191	UB(22)=	.626214399656497
W(23)=	.013963974189550	UB(23)=	.754041044869965
W(24)=	.025419145359964	UB(24)=	.863140382765184
W(25)=	0.000000000000000	UB(25)=	-.779650899866631
W(26)=	.021519808762790	UB(26)=	-.754041044869965
W(27)=	.012467227676191	UB(27)=	-.626214399656497
W(28)=	.012467227676191	UB(28)=	-.464447038235473
W(29)=	.021519808762790	UB(29)=	-.198185842870421
W(30)=	.021519808762790	UB(30)=	.198185842870421
W(31)=	.012467227676191	UB(31)=	.464447038235473
W(32)=	.012467227676191	UB(32)=	.626214399656497
W(33)=	.021519808762790	UB(33)=	.754041044869965
W(34)=	0.000000000000000	UB(34)=	-.656827300476625
W(35)=	.021519808762790	UB(35)=	-.626214399656497
W(36)=	.013963974189550	UB(36)=	-.464447038235473
W(37)=	.021519808762790	UB(37)=	-.198185842870421
W(38)=	.021519808762790	UB(38)=	.198185842870421
W(39)=	.013963974189550	UB(39)=	.464447038235473
W(40)=	.021519808762790	UB(40)=	.626214399656497
W(41)=	0.000000000000000	UB(41)=	-.504964037967035
W(42)=	.025419145359964	UB(42)=	-.464447038235473
W(43)=	.025419145359964	UB(43)=	-.198185842870421
W(44)=	.025419145359964	UB(44)=	.198185842870421
W(45)=	.025419145359964	UB(45)=	.464447038235473
W(46)=	0.000000000000000	UB(46)=	-.280277106857707
W(47)=	.046357556555418	UB(47)=	-.198185842870421
W(48)=	.046357556555418	UB(48)=	.198185842870421

N =14 IGE =1 SLABS AND SPHERES

W(1)=	0.000000000000000	UB(1)=	-1.000000000000000
W(2)=	.039326869832498	UB(2)=	-.965775002427268
W(3)=	.042397011687780	UB(3)=	-.884801870374588
W(4)=	.046220061116284	UB(4)=	-.795630155488940
W(5)=	.052449506373218	UB(5)=	-.695111745569260
W(6)=	.062746578425365	UB(6)=	-.577350269189623
W(7)=	.084331802130589	UB(7)=	-.428353041121825
W(8)=	.172528170434266	UB(8)=	-.183410256919573
W(9)=	.172528170434266	UB(9)=	.183410256919573
W(10)=	.084331802130589	UB(10)=	.428353041121825
W(11)=	.062746578425365	UB(11)=	.577350269189623
W(12)=	.052449506373218	UB(12)=	.695111745569260
W(13)=	.046220061116284	UB(13)=	.795630155488940
W(14)=	.042397011687780	UB(14)=	.884801870374588
W(15)=	.039326869832498	UB(15)=	.965775002427268

Table 8 (continued)

MOMENTS

N = 14   Q(1)= -.7105E-12

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	1.0881	2.3147	1.8418	-1.2378
2	-.0000	2.4350	1.5469	-1.8570
3	.3654	2.4875	1.2010	-2.5164
4	.9708	2.4759	.8059	-3.2146
5	1.4705	2.4032	.3629	-3.9501
6	1.8455	2.2718	-.1264	-4.7212
7	2.1212	2.0840	-.6605	-5.5265

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS = 1.8587

N =14   IGE =2   CYLINDERS

W(1)=	0.000000000000000	UR(1)=	-.983036457948888
W(2)=	.039326869832498	UR(2)=	-.965775002427268
W(3)=	.021198505843890	UR(3)=	-.884801870374588
W(4)=	.017442839180850	UR(4)=	-.795630155488940
W(5)=	.016591740719787	UR(5)=	-.695111745569260
W(6)=	.017442839180850	UR(6)=	-.577350269189623
W(7)=	.021198505843890	UR(7)=	-.428353041121825
W(8)=	.039326869832498	UR(8)=	-.183410256919573
W(9)=	.039326869832498	UR(9)=	.183410256919573
W(10)=	.021198505843890	UR(10)=	.428353041121825
W(11)=	.017442839180850	UR(11)=	.577350269189623
W(12)=	.016591740719787	UR(12)=	.695111745569260
W(13)=	.017442839180850	UR(13)=	.795630155488940
W(14)=	.021198505843890	UR(14)=	.884801870374588
W(15)=	.039326869832498	UR(15)=	.965775002427268
W(16)=	0.000000000000000	UR(16)=	-.903611460840157
W(17)=	.021198505843890	UR(17)=	-.884801870374588
W(18)=	.011334382754583	UR(18)=	-.795630155488940
W(19)=	.009633012466822	UR(19)=	-.695111745569260
W(20)=	.009633012466822	UR(20)=	-.577350269189623
W(21)=	.011334382754583	UR(21)=	-.428353041121825
W(22)=	.021198505843890	UR(22)=	-.183410256919573
W(23)=	.021198505843890	UR(23)=	.183410256919573
W(24)=	.011334382754583	UR(24)=	.428353041121825
W(25)=	.009633012466822	UR(25)=	.577350269189623
W(26)=	.009633012466822	UR(26)=	.695111745569260
W(27)=	.011334382754583	UR(27)=	.795630155488940
W(28)=	.021198505843890	UR(28)=	.884801870374588
W(29)=	0.000000000000000	UR(29)=	-.816496580927726
W(30)=	.017442839180850	UR(30)=	-.795630155488940

Table 8 (continued)

W(31)=	.009633012466822	UB(31)=	-.695111745569260
W(32)=	.008594875130020	UB(32)=	-.577350269189623
W(33)=	.009633012466822	UB(33)=	-.428353041121825
W(34)=	.017442839180850	UB(34)=	-.183410256919573
W(35)=	.017442839180850	UB(35)=	.183410256919573
W(36)=	.009633012466822	UB(36)=	.428353041121825
W(37)=	.008594875130020	UB(37)=	.577350269189623
W(38)=	.009633012466822	UB(38)=	.695111745569260
W(39)=	.017442839180850	UB(39)=	.795630155488940
W(40)=	0.000000000000000	UB(40)=	-.718901704805081
W(41)=	.016591740719787	UB(41)=	-.695111745569260
W(42)=	.009633012466822	UB(42)=	-.577350269189623
W(43)=	.009633012466822	UB(43)=	-.428353041121825
W(44)=	.016591740719787	UB(44)=	-.183410256919573
W(45)=	.016591740719787	UB(45)=	.183410256919573
W(46)=	.009633012466822	UB(46)=	.428353041121825
W(47)=	.009633012466822	UB(47)=	.577350269189623
W(48)=	.016591740719787	UB(48)=	.695111745569260
W(49)=	0.000000000000000	UB(49)=	-.605782680238256
W(50)=	.017442839180850	UB(50)=	-.577350269189623
W(51)=	.011334382754583	UB(51)=	-.428353041121825
W(52)=	.017442839180850	UB(52)=	-.183410256919573
W(53)=	.017442839180850	UB(53)=	.183410256919573
W(54)=	.011334382754583	UB(54)=	.428353041121825
W(55)=	.017442839180850	UB(55)=	.577350269189623
W(56)=	0.000000000000000	UB(56)=	-.465967434679323
W(57)=	.021198505843890	UB(57)=	-.428353041121825
W(58)=	.021198505843890	UB(58)=	-.183410256919573
W(59)=	.021198505843890	UB(59)=	.183410256919573
W(60)=	.021198505843890	UB(60)=	.428353041121825
W(61)=	0.000000000000000	UB(61)=	-.259381272813998
W(62)=	.039326869832498	UB(62)=	-.183410256919573
W(63)=	.039326869832498	UB(63)=	.183410256919573

N =16 IGE =1 SLABS AND SPHERES

W(1)=	0.000000000000000	UB(1)=	-1.000000000000000
W(2)=	.034150743173670	UB(2)=	-.970141999915473
W(3)=	.036379491801739	UB(3)=	-.900512968892091
W(4)=	.038944135405336	UB(4)=	-.825028311202534
W(5)=	.042967202111948	UB(5)=	-.741902838266956
W(6)=	.048915735831746	UB(6)=	-.648203616598540
W(7)=	.058595584986041	UB(7)=	-.538438516187579
W(8)=	.078806650698751	UB(8)=	-.399579958027353
W(9)=	.161240455990769	UB(9)=	-.171500000000000
W(10)=	.161240455990769	UB(10)=	.171500000000000
W(11)=	.078806650698751	UB(11)=	.399579958027353
W(12)=	.058595584986041	UB(12)=	.538438516187579
W(13)=	.048915735831746	UB(13)=	.648203616598540
W(14)=	.042967202111948	UB(14)=	.741902838266956
W(15)=	.038944135405336	UB(15)=	.825028311202534
W(16)=	.036379491801739	UB(16)=	.900512968892091
W(17)=	.034150743173670	UB(17)=	.970141999915473

Table 8 (continued)

MOMENTS

N = 16 Q(1)= -.7105E-12

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	.9194	2.4357	1.9023	-1.2074
2	-.0000	2.5364	1.6414	-1.7481
3	.3796	2.5848	1.3418	-2.3192
4	.9441	2.5837	1.0047	-2.9196
5	1.4132	2.5352	.6311	-3.5485
6	1.7788	2.4413	.2221	-4.2047
7	2.0629	2.3035	-.2213	-4.8873
8	2.2794	2.1234	-.6982	-5.5952

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS = 1.9216

N =16 IGE =2 CYLINDERS

W(1)=	0.000000000000000	UB(1)=	-.985184119847656
W(2)=	.034150743173670	UB(2)=	-.970141999915473
W(3)=	.018189745900870	UB(3)=	-.900512968892091
W(4)=	.014693679924853	UB(4)=	-.825028311202534
W(5)=	.013586058995992	UB(5)=	-.741902838266956
W(6)=	.013586058995992	UB(6)=	-.648203616598540
W(7)=	.014693679924853	UB(7)=	-.538438516187579
W(8)=	.018189745900870	UB(8)=	-.399579958027353
W(9)=	.034150743173670	UB(9)=	-.171500000000000
W(10)=	.034150743173670	UB(10)=	.171500000000000
W(11)=	.018189745900870	UB(11)=	.399579958027353
W(12)=	.014693679924853	UB(12)=	.538438516187579
W(13)=	.013586058995992	UB(13)=	.648203616598540
W(14)=	.013586058995992	UB(14)=	.741902838266956
W(15)=	.014693679924853	UB(15)=	.825028311202534
W(16)=	.018189745900870	UB(16)=	.900512968892091
W(17)=	.034150743173670	UB(17)=	.970141999915473
W(18)=	0.000000000000000	UB(18)=	-.916698345772947
W(19)=	.018189745900870	UB(19)=	-.900512968892091
W(20)=	.009556775555629	UB(20)=	-.825028311202534
W(21)=	.007897542059982	UB(21)=	-.741902838266956
W(22)=	.007518523665789	UB(22)=	-.648203616598540
W(23)=	.007897542059982	UB(23)=	-.538438516187579
W(24)=	.009556775555629	UB(24)=	-.399579958027353
W(25)=	.018189745900870	UB(25)=	-.171500000000000
W(26)=	.018189745900870	UB(26)=	.171500000000000
W(27)=	.009556775555629	UB(27)=	.399579958027353
W(28)=	.007897542059982	UB(28)=	.538438516187579
W(29)=	.007518523665789	UB(29)=	.648203616598540
W(30)=	.007897542059982	UB(30)=	.741902838266956

Table 8 (continued)

W(31)=	.00955677555629	UB(31)=	.825028311202534
W(32)=	.01818974590087 0	UB(32)=	.900512968892091
W(33)=	0.00000000000000 0	UB(33)=	-.842664799481806
W(34)=	.01469367992485 3	UB(34)=	-.825028311202534
W(35)=	.00789754205998 2	UB(35)=	-.741902838266956
W(36)=	.00670657050818 5	UB(36)=	-.648203616598540
W(37)=	.00670657050818 5	UB(37)=	-.538438516187579
W(38)=	.00789754205998 2	UB(38)=	-.399579958027353
W(39)=	.01469367992485 3	UB(39)=	-.171500000000000
W(40)=	.01469367992485 3	UB(40)=	.171500000000000
W(41)=	.00789754205998 2	UB(41)=	.399579958027353
W(42)=	.00670657050818 5	UB(42)=	.538438516187579
W(43)=	.00670657050818 5	UB(43)=	.648203616598540
W(44)=	.00789754205998 2	UB(44)=	.741902838266956
W(45)=	.01469367992485 3	UB(45)=	.825028311202534
W(46)=	0.00000000000000 0	UB(46)=	-.761467052096524
W(47)=	.01358605899599 2	UB(47)=	-.741902838266956
W(48)=	.00751852366578 9	UB(48)=	-.648203616598540
W(49)=	.00670657050818 5	UB(49)=	-.538438516187579
W(50)=	.00751852366578 9	UB(50)=	-.399579958027353
W(51)=	.01358605899599 2	UB(51)=	-.171500000000000
W(52)=	.01358605899599 2	UB(52)=	.171500000000000
W(53)=	.00751852366578 9	UB(53)=	.399579958027353
W(54)=	.00670657050818 5	UB(54)=	.538438516187579
W(55)=	.00751852366578 9	UB(55)=	.648203616598540
W(56)=	.01358605899599 2	UB(56)=	.741902838266956
W(57)=	0.00000000000000 0	UB(57)=	-.670507403815520
W(58)=	.01358605899599 2	UB(58)=	-.648203616598540
W(59)=	.00789754205998 2	UB(59)=	-.538438516187579
W(60)=	.00789754205998 2	UB(60)=	-.399579958027353
W(61)=	.01358605899599 2	UB(61)=	-.171500000000000
W(62)=	.01358605899599 2	UB(62)=	.171500000000000
W(63)=	.00789754205998 2	UB(63)=	.399579958027353
W(64)=	.00789754205998 2	UB(64)=	.538438516187579
W(65)=	.01358605899599 2	UB(65)=	.648203616598540
W(66)=	0.00000000000000 0	UB(66)=	-.565091395894765
W(67)=	.01469367992485 3	UB(67)=	-.538438516187579
W(68)=	.00955677555562 9	UB(68)=	-.399579958027353
W(69)=	.01469367992485 3	UB(69)=	-.171500000000000
W(70)=	.01469367992485 3	UB(70)=	.171500000000000
W(71)=	.00955677555562 9	UB(71)=	.399579958027353
W(72)=	.01469367992485 3	UB(72)=	.538438516187579
W(73)=	0.00000000000000 0	UB(73)=	-.434829153642156
W(74)=	.01818974590087 0	UB(74)=	-.399579958027353
W(75)=	.01818974590087 0	UB(75)=	-.171500000000000
W(76)=	.01818974590087 0	UB(76)=	.171500000000000
W(77)=	.01818974590087 0	UB(77)=	.399579958027353
W(78)=	0.00000000000000 0	UB(78)=	-.242537625947000
W(79)=	.03415074317367 0	UB(79)=	-.171500000000000
W(80)=	.03415074317367 0	UB(80)=	.171500000000000

Table 8 (continued)

N =20 IGE =1 SLABS AND SPHERES

W(1)=	0.000000000000000	UB(1)=	-1.000000000000000
W(2)=	.027037345002054	UB(2)=	-.976219829117284
W(3)=	.028355452566102	UB(3)=	-.921806117281200
W(4)=	.029686530720196	UB(4)=	-.863972152880113
W(5)=	.031753818741709	UB(5)=	-.801978331407749
W(6)=	.034512680197183	UB(6)=	-.734772486653391
W(7)=	.038250093272212	UB(7)=	-.660766199376202
W(8)=	.043629782027811	UB(8)=	-.577350269189623
W(9)=	.052316254897566	UB(9)=	-.479640173910180
W(10)=	.070406017007737	UB(10)=	-.356056258930870
W(11)=	.144052025567428	UB(11)=	-.153288690447485
W(12)=	.144052025567428	UB(12)=	.153288690447485
W(13)=	.070406017007737	UB(13)=	.356056258930870
W(14)=	.052316254897566	UB(14)=	.479640173910180
W(15)=	.043629782027811	UB(15)=	.577350269189623
W(16)=	.038250093272212	UB(16)=	.660766199376202
W(17)=	.034512680197183	UB(17)=	.734772486653391
W(18)=	.031753818741709	UB(18)=	.801978331407749
W(19)=	.029686530720196	UB(19)=	.863972152880113
W(20)=	.028355452566102	UB(20)=	.921806117281200
W(21)=	.027037345002054	UB(21)=	.976219829117284

MOMENTS

N = 20 Q(1)= -.1421E-11

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	.6851	2.6214	2.0175	-1.1230
2	-.0000	2.6981	1.8068	-1.5536
3	.3846	2.7411	1.5716	-2.0033
4	.8850	2.7520	1.3126	-2.4715
5	1.3059	2.7319	1.0302	-2.9579
6	1.6499	2.6821	.7251	-3.4619
7	1.9343	2.6034	.3978	-3.9829
8	2.1688	2.4969	.0489	-4.5205
9	2.3592	2.3633	-.3213	-5.0741
10	2.5092	2.2032	-.7121	-5.6432

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS = 2.0125

Table 8 (continued)

N =24   IGE =1   SLABS AND SPHERES

W(1)=	0.000000000000000	UB(1)=	-1.000000000000000
W(2)=	.022377993115599	UB(2)=	-.980248122997860
W(3)=	.023240826176660	UB(3)=	-.935580550688272
W(4)=	.024014988139755	UB(4)=	-.888670665101255
W(5)=	.025260130757786	UB(5)=	-.839142499934816
W(6)=	.026850694116718	UB(6)=	-.786501569853609
W(7)=	.028850108118882	UB(7)=	-.730074861618679
W(8)=	.031416971700271	UB(8)=	-.668904991574195
W(9)=	.034853489567805	UB(9)=	-.601546732962799
W(10)=	.039778170417273	UB(10)=	-.525626346489176
W(11)=	.047715262911175	UB(11)=	-.436700859065905
W(12)=	.064231846597990	UB(12)=	-.324240997553115
W(13)=	.131409518380084	UB(13)=	-.139845660210033
W(14)=	.131409518380084	UB(14)=	.139845660210033
W(15)=	.064231846597990	UB(15)=	.324240997553115
W(16)=	.047715262911175	UB(16)=	.436700859065905
W(17)=	.039778170417273	UB(17)=	.525626346489176
W(18)=	.034853489567805	UB(18)=	.601546732962799
W(19)=	.031416971700271	UB(19)=	.668904991574195
W(20)=	.028850108118882	UB(20)=	.730074861618679
W(21)=	.026850694116718	UB(21)=	.786501569853609
W(22)=	.025260130757786	UB(22)=	.839142499934816
W(23)=	.024014988139755	UB(23)=	.888670665101255
W(24)=	.023240826176660	UB(24)=	.935580550688272
W(25)=	.022377993115599	UB(25)=	.980248122997860

MOMENTS

N = 24   Q(1)= -.1421E-11

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	.5317	2.7567	2.1187	-1.0326
2	-.0000	2.8196	1.9429	-1.3897
3	.3756	2.8590	1.7502	-1.7599
4	.8267	2.8757	1.5409	-2.1430
5	1.2099	2.8707	1.3155	-2.5386
6	1.5322	2.8446	1.0743	-2.9465
7	1.8076	2.7981	.8175	-3.3663
8	2.0437	2.7317	.5454	-3.7978
9	2.2452	2.6460	.2585	-4.2407
10	2.4152	2.5413	-.0431	-4.6947
11	2.5559	2.4183	-.3590	-5.1594
12	2.6692	2.2773	-.6889	-5.6347

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS =   2.0778

Table 8 (continued)

N =28   IGE =1   SLABS AND SPHERES

W(1)=	0.000000000000000	UB(1)=	-1.000000000000000
W(2)=	.019089083961131	UB(2)=	-.983113649643620
W(3)=	.019692950954119	UB(3)=	-.945226596899854
W(4)=	.020173737521405	UB(4)=	-.905756143152313
W(5)=	.020997480305107	UB(5)=	-.864485432051591
W(6)=	.022029006299018	UB(6)=	-.821143065245458
W(7)=	.023275663287697	UB(7)=	-.775381715654930
W(8)=	.024788751099092	UB(8)=	-.726744574347315
W(9)=	.026660780477876	UB(9)=	-.674609922632730
W(10)=	.029049244962621	UB(10)=	-.618093374083408
W(11)=	.032238140169552	UB(11)=	-.555859955795604
W(12)=	.036801966729094	UB(12)=	-.485717162377824
W(13)=	.044152724006060	UB(13)=	-.403561684503922
W(14)=	.059444810691657	UB(14)=	-.299671327575593
W(15)=	.121605659535571	UB(15)=	-.129397743188183
W(16)=	.121605659535571	UB(16)=	.129397743188183
W(17)=	.059444810691657	UB(17)=	.299671327575593
W(18)=	.044152724006060	UB(18)=	.403561684503922
W(19)=	.036801966729094	UB(19)=	.485717162377824
W(20)=	.032238140169552	UB(20)=	.555859955795604
W(21)=	.029049244962621	UB(21)=	.618093374083408
W(22)=	.026660780477876	UB(22)=	.674609922632730
W(23)=	.024788751099092	UB(23)=	.726744574347315
W(24)=	.023275663287697	UB(24)=	.775381715654930
W(25)=	.022029006299018	UB(25)=	.821143065245458
W(26)=	.020997480305107	UB(26)=	.864485432051591
W(27)=	.020173737521405	UB(27)=	.905756143152313
W(28)=	.019692950954119	UB(28)=	.945226596899854
W(29)=	.019089083961131	UB(29)=	.983113649643620

MOMENTS

N =   28   Q(1)= -.2132E-11

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	.4244	2.8605	2.2070	-.9451
2	-.0000	2.9145	2.0567	-1.2497
3	.3615	2.9511	1.8940	-1.5639
4	.7732	2.9709	1.7192	-1.8875
5	1.1255	2.9746	1.5326	-2.2204
6	1.4275	2.9625	1.3342	-2.5623
7	1.6910	2.9351	1.1243	-2.9130
8	1.9222	2.8928	.9031	-3.2724
9	2.1251	2.8360	.6709	-3.6403
10	2.3023	2.7649	.4277	-4.0165
11	2.4558	2.6800	.1738	-4.4009
12	2.5871	2.5815	-.0906	-4.7931
13	2.6977	2.4696	-.3653	-5.1931
14	2.7885	2.3447	-.6502	-5.6007

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS =   2.1268

Table 8 (continued)

N =32 IGE =1 SLABS AND SPHERES

W(1)=	0.000000000000000	UR(1)=	-1.000000000000000
W(2)=	.016643326757938	UR(2)=	-.985256136077826
W(3)=	.017086447816649	UR(3)=	-.952360237328524
W(4)=	.017396546590761	UR(4)=	-.918286659823540
W(5)=	.017976469903380	UR(5)=	-.882899064205702
W(6)=	.018697204851399	UR(6)=	-.846032579479502
W(7)=	.019548370100563	UR(7)=	-.807484670756150
W(8)=	.020548030821511	UR(8)=	-.767001865363834
W(9)=	.021732162935595	UR(9)=	-.724259780353037
W(10)=	.023157934689386	UR(10)=	-.678831788738908
W(11)=	.024915526382391	UR(11)=	-.630137259149066
W(12)=	.027153927401173	UR(12)=	-.577350269189623
W(13)=	.030139622538597	UR(13)=	-.519224133971793
W(14)=	.034410426410850	UR(14)=	-.453711438321973
W(15)=	.041287309670913	UR(15)=	-.376980685486174
W(16)=	.055591822573321	UR(16)=	-.279955005661728
W(17)=	.113714870555570	UR(17)=	-.120975919754679
W(18)=	.113714870555570	UR(18)=	.120975919754679
W(19)=	.055591822573321	UR(19)=	.279955005661728
W(20)=	.041287309670913	UR(20)=	.376980685486174
W(21)=	.034410426410850	UR(21)=	.453711438321973
W(22)=	.030139622538597	UR(22)=	.519224133971793
W(23)=	.027153927401173	UR(23)=	.577350269189623
W(24)=	.024915526382391	UR(24)=	.630137259149066
W(25)=	.023157934689386	UR(25)=	.678831788738908
W(26)=	.021732162935595	UR(26)=	.724259780353037
W(27)=	.020548030821511	UR(27)=	.767001865363834
W(28)=	.019548370100563	UR(28)=	.807484670756150
W(29)=	.018697204851399	UR(29)=	.846032579479502
W(30)=	.017976469903380	UR(30)=	.882899064205702
W(31)=	.017396546590761	UR(31)=	.918286659823540
W(32)=	.017086447816649	UR(32)=	.952360237328524
W(33)=	.016643326757938	UR(33)=	.985256136077826

MOMENTS

N = 32 Q(1)= -.2132E-11

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	.3457	2.9437	2.2850	-.8624
2	-.0000	2.9914	2.1541	-1.1277
3	.3459	3.0258	2.0138	-1.4004
4	.7250	3.0472	1.8642	-1.6802
5	1.0515	3.0561	1.7054	-1.9671
6	1.3352	3.0528	1.5377	-2.2609
7	1.5861	3.0376	1.3611	-2.5616
8	1.8099	3.0108	1.1758	-2.8689
9	2.0098	2.9726	.9820	-3.1828
10	2.1881	2.9233	.7796	-3.5032
11	2.3466	2.8631	.5690	-3.8299
12	2.4866	2.7923	.3501	-4.1629

Table 8 (continued)

13	2.6093	2.7110	.1233	-4.5019
14	2.7156	2.6195	-.1116	-4.8469
15	2.8064	2.5178	-.3542	-5.1977
16	2.8822	2.4063	-.6045	-5.5542

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS = 2.1645

N =40 IGE =1 SLABS AND SPHERES

W(1)=	0.000000000000000	UB(1)=	-1.000000000000000
W(2)=	.013248768917978	UB(2)=	-.988245614367752
W(3)=	.013511872300549	UB(3)=	-.962207287016053
W(4)=	.013645718156071	UB(4)=	-.935444456959630
W(5)=	.013970258687021	UB(5)=	-.907893055886102
W(6)=	.014375504662528	UB(6)=	-.879478976323995
W(7)=	.014843178512361	UB(7)=	-.850115720749599
W(8)=	.015373254381062	UB(8)=	-.819701291651608
W(9)=	.015972929940773	UB(9)=	-.788113999625061
W(10)=	.016654518271053	UB(10)=	-.755206690432967
W(11)=	.017435828188347	UB(11)=	-.720798594715902
W(12)=	.018341971065111	UB(12)=	-.684663481583559
W(13)=	.019408827883381	UB(13)=	-.646511834295264
W(14)=	.020689293919767	UB(14)=	-.605962887274071
W(15)=	.022264949977925	UB(15)=	-.562498435218473
W(16)=	.024269565800262	UB(16)=	-.515381371891664
W(17)=	.026941775687476	UB(17)=	-.463499112580205
W(18)=	.030762751987602	UB(18)=	-.405024562504966
W(19)=	.036914019948345	UB(19)=	-.336538801777808
W(20)=	.049708142380843	UB(20)=	-.249943661595459
W(21)=	.101666869331541	UB(21)=	-.108098579275809
W(22)=	.101666869331541	UB(22)=	.108098579275809
W(23)=	.049708142380843	UB(23)=	.249943661595459
W(24)=	.036914019948345	UB(24)=	.336538801777808
W(25)=	.030762751987602	UB(25)=	.405024562504966
W(26)=	.026941775687476	UB(26)=	.463499112580205
W(27)=	.024269565800262	UB(27)=	.515381371891664
W(28)=	.022264949977925	UB(28)=	.562498435218473
W(29)=	.020689293919767	UB(29)=	.605962887274071
W(30)=	.019408827883381	UB(30)=	.646511834295264
W(31)=	.018341971065111	UB(31)=	.684663481583559
W(32)=	.017435828188347	UB(32)=	.720798594715902
W(33)=	.016654518271053	UB(33)=	.755206690432967
W(34)=	.015972929940773	UB(34)=	.788113999625061
W(35)=	.015373254381062	UB(35)=	.819701291651608
W(36)=	.014843178512361	UB(36)=	.850115720749599
W(37)=	.014375504662528	UB(37)=	.879478976323995
W(38)=	.013970258687021	UB(38)=	.907893055886102
W(39)=	.013645718156071	UB(39)=	.935444456959630
W(40)=	.013511872300549	UB(40)=	.962207287016053
W(41)=	.013248768917978	UB(41)=	.988245614367752

Table 8 (continued)

MOMENTS

N = 40 Q(1)= -.3553E-11

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	.2395	3.0713	2.4185	-.7114
2	-.0000	3.1107	2.3151	-.9218
3	.3156	3.1416	2.2057	-1.1369
4	.6439	3.1640	2.0903	-1.3565
5	.9291	3.1783	1.9690	-1.5807
6	1.1815	3.1847	1.8420	-1.8094
7	1.4091	3.1832	1.7092	-2.0424
8	1.6160	3.1741	1.5708	-2.2799
9	1.8050	3.1574	1.4268	-2.5216
10	1.9778	3.1335	1.2773	-2.7675
11	2.1358	3.1023	1.1224	-3.0175
12	2.2801	3.0640	.9621	-3.2717
13	2.4116	3.0188	.7965	-3.5299
14	2.5310	2.9667	.6256	-3.7921
15	2.6390	2.9079	.4496	-4.0582
16	2.7360	2.8424	.2685	-4.3281
17	2.8225	2.7703	.0823	-4.6019
18	2.8989	2.6918	-.1088	-4.8793
19	2.9657	2.6070	-.3048	-5.1604
20	3.0230	2.5158	-.5057	-5.4451

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS = 2.2199

N =48 IGE =1 SLABS AND SPHERES

W(1)=	0.000000000000000	UB(1)=	-1.000000000000000
W(2)=	.011004500018045	UB(2)=	-.990231726256908
W(3)=	.011175073723283	UB(3)=	-.968683906384758
W(4)=	.011227862781370	UB(4)=	-.946645735896983
W(5)=	.011429416333522	UB(5)=	-.924082132764749
W(6)=	.011686132073598	UB(6)=	-.900953620836361
W(7)=	.011980019417486	UB(7)=	-.877215518388301
W(8)=	.012307366916966	UB(8)=	-.852816923204713
W(9)=	.012669207657044	UB(9)=	-.827699428118361
W(10)=	.013069005521269	UB(10)=	-.801795473989824
W(11)=	.013512086032125	UB(11)=	-.775026206597989
W(12)=	.014005691553957	UB(12)=	-.747298641586330
W(13)=	.014559396718087	UB(13)=	-.718501843087321
W(14)=	.015185873794486	UB(14)=	-.688501061089486
W(15)=	.015902133357104	UB(15)=	-.657133301041370
W(16)=	.016731525690443	UB(16)=	-.624190527747089
W(17)=	.017707060623145	UB(17)=	-.589409410963462
W(18)=	.018877182513209	UB(18)=	-.552442840966737
W(19)=	.020316463516794	UB(19)=	-.512818419461023
W(20)=	.022147078324144	UB(20)=	-.469864203937274
W(21)=	.024586900004138	UB(21)=	-.422565981764638
W(22)=	.028075179066400	UB(22)=	-.369258239918647
W(23)=	.033690465928980	UB(23)=	-.306824520778443
W(24)=	.045369627928145	UB(24)=	-.227883578508908
W(25)=	.092784750506257	UB(25)=	-.098592921435210
W(26)=	.092784750506257	UB(26)=	.098592921435210
W(27)=	.045369627928145	UB(27)=	.227883578508908

Table 8 (continued)

W(28)=	.033690465928980	UR(28)=	.306824520778443
W(29)=	.028075179066400	UR(29)=	.369258239918647
W(30)=	.024586900004138	UR(30)=	.422565981764638
W(31)=	.022147078324144	UR(31)=	.469864203937274
W(32)=	.020316463516794	UR(32)=	.512818419461023
W(33)=	.018877182513209	UR(33)=	.552442840966737
W(34)=	.017707060623145	UR(34)=	.589409410963462
W(35)=	.016731525690443	UR(35)=	.624190527747089
W(36)=	.015902133357104	UR(36)=	.657133301641370
W(37)=	.015185873794486	UR(37)=	.688501661089486
W(38)=	.014559396718087	UR(38)=	.718501843087321
W(39)=	.014005691553957	UB(39)=	.747298641586330
W(40)=	.013512086032125	UR(40)=	.775026206597989
W(41)=	.013069005521269	UB(41)=	.801795473989824
W(42)=	.012669207657044	UB(42)=	.827699428118361
W(43)=	.012307366916966	UR(43)=	.852816923204713
W(44)=	.011980019417486	UR(44)=	.877215518388301
W(45)=	.011686132073598	UR(45)=	.900953620836361
W(46)=	.011429416333522	UR(46)=	.924082132764749
W(47)=	.011227862781370	UR(47)=	.946645735896983
W(48)=	.011175073723283	UR(48)=	.968683906384758
W(49)=	.011004500018045	UR(49)=	.990231726456908

MOMENTS

N = 48 Q(1)= -.4263E-11

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	.1722	3.1675	2.5312	-.5764
2	-.0000	3.2016	2.4462	-.7503
3	.2888	3.2297	2.3571	-.9274
4	.5790	3.2520	2.2638	-1.1077
5	.8328	3.2685	2.1664	-1.2912
6	1.0601	3.2794	2.0650	-1.4778
7	1.2674	3.2848	1.9595	-1.6675
8	1.4583	3.2848	1.8501	-1.8602
9	1.6349	3.2794	1.7368	-2.0560
10	1.7987	3.2688	1.6197	-2.2547
11	1.9508	3.2530	1.4986	-2.4563
12	2.0922	3.2322	1.3738	-2.6609
13	2.2236	3.2064	1.2453	-2.8683
14	2.3454	3.1757	1.1130	-3.0786
15	2.4583	3.1401	.9770	-3.2917
16	2.5627	3.0997	.8374	-3.5075
17	2.6588	3.0547	.6942	-3.7260
18	2.7472	3.0049	.5475	-3.9473
19	2.8280	2.9506	.3972	-4.1712
20	2.9015	2.8917	.2434	-4.3977
21	2.9680	2.8284	.0862	-4.6268
22	3.0276	2.7606	-.0744	-4.8584
23	3.0806	2.6885	-.2384	-5.0926
24	3.1272	2.6120	-.4057	-5.3292

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS = 2.2597

Table 8 (continued)

N =56 IGE =1 SLABS AND SPHERES

W(1)=	0.000000000000000	UR(1)=	-1.000000000000000
W(2)=	.009410493374558	UR(2)=	-.991646736033182
W(3)=	.009527733116603	UR(3)=	-.973267936381308
W(4)=	.009538827066233	UR(4)=	-.954535333495102
W(5)=	.009672557286623	UR(5)=	-.935427072133592
W(6)=	.009848000590556	UR(6)=	-.915921479547208
W(7)=	.010048743524827	UR(7)=	-.895990727406602
W(8)=	.010270333479242	UR(8)=	-.875606424428970
W(9)=	.010512039845773	UR(9)=	-.854736121504150
W(10)=	.010774762258558	UR(10)=	-.833343305190979
W(11)=	.011060382690950	UR(11)=	-.811386647172171
W(12)=	.011371579376893	UR(12)=	-.788819065509934
W(13)=	.011711834736597	UR(13)=	-.765586536594419
W(14)=	.012085558966283	UR(14)=	-.741626571744369
W(15)=	.012498316256336	UR(15)=	-.716866234955990
W(16)=	.012957170782739	UR(16)=	-.691219520646406
W(17)=	.013471202656450	UR(17)=	-.664583819111872
W(18)=	.014052279882001	UR(18)=	-.636835048916115
W(19)=	.014716239847377	UR(19)=	-.607820784796523
W(20)=	.015484759360071	UR(20)=	-.577350269189626
W(21)=	.016388439531903	UR(21)=	-.545179383539036
W(22)=	.017472158345081	UR(22)=	-.510987071400706
W(23)=	.018804972955694	UR(23)=	-.474336393334248
W(24)=	.020500004530006	UR(24)=	-.434605845501430
W(25)=	.022758965151359	UR(25)=	-.390857349741161
W(26)=	.025988511907099	UR(26)=	-.341550427827801
W(27)=	.031187167731323	UR(27)=	-.283802610368589
W(28)=	.041999826272762	UR(28)=	-.210786974347789
W(29)=	.085887138476101	UR(29)=	-.091205128459786
W(30)=	.085887138476101	UR(30)=	.091205128459786
W(31)=	.041999826272762	UR(31)=	.210786974347789
W(32)=	.031187167731323	UR(32)=	.283802610368589
W(33)=	.025988511907099	UR(33)=	.341550427827801
W(34)=	.022758965151359	UR(34)=	.390857349741161
W(35)=	.020500004530006	UR(35)=	.434605845501430
W(36)=	.018804972955694	UR(36)=	.474336393334248
W(37)=	.017472158345081	UR(37)=	.510987071400706
W(38)=	.016388439531903	UR(38)=	.545179383539036
W(39)=	.015484759360071	UR(39)=	.577350269189626
W(40)=	.014716239847377	UR(40)=	.607820784796523
W(41)=	.014052279882001	UR(41)=	.636835048916115
W(42)=	.013471202656450	UR(42)=	.664583819111872
W(43)=	.012957170782739	UR(43)=	.691219520646406
W(44)=	.012498316256336	UR(44)=	.716866234955990
W(45)=	.012085558966283	UR(45)=	.741626571744369
W(46)=	.011711834736597	UR(46)=	.765586536594419
W(47)=	.011371579376893	UR(47)=	.788819065509934
W(48)=	.011060382690950	UR(48)=	.811386647172171
W(49)=	.010774762258558	UR(49)=	.833343305190979
W(50)=	.010512039845773	UR(50)=	.854736121504150
W(51)=	.010270333479242	UR(51)=	.875606424428970
W(52)=	.010048743524827	UR(52)=	.895990727406602
W(53)=	.009848000590556	UR(53)=	.915921479547208
W(54)=	.009672557286623	UR(54)=	.935427072133592
W(55)=	.009538827066233	UR(55)=	.954535333495102
W(56)=	.009527733116603	UR(56)=	.973267936381308
W(57)=	.009410493374558	UR(57)=	.991646736033182

Table 8 (continued)

MOMENTS

N = 56 Q(1)= -.4263E-11

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	.1264	3.2448	2.6297	-.4537
2	-.0000	3.2752	2.5579	-.6016
3	.2657	3.3011	2.4831	-.7519
4	.5263	3.3228	2.4051	-.9045
5	.7553	3.3401	2.3242	-1.0595
6	.9621	3.3533	2.2403	-1.2167
7	1.1523	3.3624	2.1535	-1.3763
8	1.3288	3.3674	2.0637	-1.5381
9	1.4937	3.3685	1.9710	-1.7021
10	1.6480	3.3656	1.8754	-1.8684
11	1.7929	3.3588	1.7770	-2.0368
12	1.9289	3.3483	1.6757	-2.2074
13	2.0567	3.3339	1.5717	-2.3801
14	2.1768	3.3159	1.4649	-2.5550
15	2.2896	3.2942	1.3553	-2.7319
16	2.3954	3.2689	1.2430	-2.9110
17	2.4946	3.2401	1.1280	-3.0921
18	2.5874	3.2077	1.0103	-3.2752
19	2.6742	3.1718	.8900	-3.4603
20	2.7550	3.1325	.7671	-3.6475
21	2.8302	3.0898	.6415	-3.8366
22	2.8998	3.0437	.5134	-4.0276
23	2.9640	2.9943	.3827	-4.2205
24	3.0231	2.9415	.2495	-4.4154
25	3.0771	2.8856	.1138	-4.6121
26	3.1261	2.8264	-.0244	-4.8107
27	3.1704	2.7640	-.1651	-5.0111
28	3.2099	2.6984	-.3082	-5.2134

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS = 2.2905

Table 8 (continued)

N =64 IGE =1 SLABS AND SPHERES

W(1)=	0.0000000000000000	UB(1)=	-1.000000000000000
W(2)=	.008219876939752	UB(2)=	-.992705835079047
W(3)=	.008303839861379	UB(3)=	-.976683225505518
W(4)=	.008291952400885	UB(4)=	-.960393341796856
W(5)=	.008384734563119	UB(5)=	-.943822345016052
W(6)=	.008511097318607	UB(6)=	-.926955159074843
W(7)=	.008656256205577	UB(7)=	-.909775310128463
W(8)=	.008815786164431	UB(8)=	-.892264738126087
W(9)=	.008988398854562	UB(9)=	-.874403574379180
W(10)=	.009174084194021	UB(10)=	-.856169877343834
W(11)=	.009373503238252	UB(11)=	-.837539316602406
W(12)=	.009587769435679	UB(12)=	-.818484792063177
W(13)=	.009818378314971	UB(13)=	-.798975971367458
W(14)=	.010067208880779	UB(14)=	-.778978722948477
W(15)=	.010336567762431	UB(15)=	-.758454414444476
W(16)=	.010629269509619	UB(16)=	-.737359035188543
W(17)=	.010948757908892	UB(17)=	-.715642085653197
W(18)=	.011299275245663	UB(18)=	-.693245153421163
W(19)=	.011686102764158	UB(19)=	-.670100060234141
W(20)=	.012115900163404	UB(20)=	-.646126410781722
W(21)=	.012597197141915	UB(21)=	-.621228288709990
W(22)=	.013141120263043	UB(22)=	-.595289706510549
W(23)=	.013762501691879	UB(23)=	-.568168181669201
W(24)=	.014481631434089	UB(24)=	-.539685399696115
W(25)=	.015327143496272	UB(25)=	-.509613165674740
W(26)=	.016341022606168	UB(26)=	-.477651365132457
W(27)=	.017587869672862	UB(27)=	-.443391559004873
W(28)=	.019173499951486	UB(28)=	-.406252781628192
W(29)=	.021286594515999	UB(29)=	-.365358140137200
W(30)=	.024307537831514	UB(30)=	-.319267628406617
W(31)=	.029170350507264	UB(31)=	-.265286762828937
W(32)=	.039284579404757	UB(32)=	-.197033790290216
W(33)=	.080330191756567	UB(33)=	-.085250000000000
W(34)=	.080330191756567	UB(34)=	.085250000000000
W(35)=	.039284579404757	UB(35)=	.197033790290216
W(36)=	.029170350507264	UB(36)=	.265286762828937
W(37)=	.024307537831514	UB(37)=	.319267628406617
W(38)=	.021286594515999	UB(38)=	.365358140137200
W(39)=	.019173499951486	UB(39)=	.406252781628192
W(40)=	.017587869672862	UB(40)=	.443391559004873
W(41)=	.016341022606168	UB(41)=	.477651365132457
W(42)=	.015327143496272	UB(42)=	.509613165674740
W(43)=	.014481631434089	UB(43)=	.539685399696115
W(44)=	.013762501691879	UB(44)=	.568168181669201
W(45)=	.013141120263043	UB(45)=	.595289706510549
W(46)=	.012597197141915	UB(46)=	.621228288709990
W(47)=	.012115900163404	UB(47)=	.646126410781722
W(48)=	.011686102764158	UB(48)=	.670100060234141
W(49)=	.011299275245663	UB(49)=	.693245153421163
W(50)=	.010948757908892	UB(50)=	.715642085653197
W(51)=	.010629269509619	UB(51)=	.737359035188543
W(52)=	.010336567762431	UB(52)=	.758454414444476
W(53)=	.010067208880779	UB(53)=	.778978722948477
W(54)=	.009818378314971	UB(54)=	.798975971367458
W(55)=	.009587769435679	UB(55)=	.818484792063177

Table 8 (continued)

W(56)=	.009373503238252	UB(56)=	.837539316602406
W(57)=	.009174084194021	UB(57)=	.856169877343834
W(58)=	.008988398854562	UB(58)=	.874403574379180
W(59)=	.008815786164431	UB(59)=	.892264738126087
W(60)=	.008656256205577	UB(60)=	.909775310128463
W(61)=	.008511097318607	UB(61)=	.926955159074843
W(62)=	.008384734563119	UB(62)=	.943822345016052
W(63)=	.008291952400885	UB(63)=	.960393341796856
W(64)=	.008303839861379	UB(64)=	.976683225505518
W(65)=	.008219876939752	UB(65)=	.992705835079047

MOMENTS

N = 64 Q(1)= -.5684E-11

I	Q(I+1)	Q(I+K2+1)	Q(I+2*K2+1)	Q(I+3*K2+1)
1	.0938	3.3098	2.7180	-.3408
2	-.0000	3.3373	2.6560	-.4692
3	.2460	3.3615	2.5918	-.5995
4	.4828	3.3824	2.5252	-.7316
5	.6916	3.3999	2.4563	-.8655
6	.8814	3.4142	2.3851	-1.0012
7	1.0571	3.4254	2.3116	-1.1386
8	1.2212	3.4334	2.2358	-1.2777
9	1.3754	3.4383	2.1579	-1.4186
10	1.5207	3.4401	2.0777	-1.5612
11	1.6581	3.4390	1.9953	-1.7055
12	1.7880	3.4348	1.9107	-1.8514
13	1.9111	3.4278	1.8239	-1.9990
14	2.0277	3.4178	1.7350	-2.1483
15	2.1381	3.4050	1.6440	-2.2992
16	2.2428	3.3893	1.5508	-2.4517
17	2.3419	3.3709	1.4556	-2.6058
18	2.4357	3.3496	1.3582	-2.7615
19	2.5244	3.3257	1.2588	-2.9188
20	2.6082	3.2990	1.1574	-3.0777
21	2.6872	3.2697	1.0539	-3.2380
22	2.7616	3.2377	.9484	-3.4000
23	2.8316	3.2031	.8409	-3.5634
24	2.8972	3.1659	.7314	-3.7283
25	2.9586	3.1261	.6200	-3.8947
26	3.0159	3.0837	.5065	-4.0626
27	3.0692	3.0389	.3912	-4.2319
28	3.1186	2.9915	.2739	-4.4027
29	3.1642	2.9417	.1547	-4.5749
30	3.2060	2.8894	.0337	-4.7485
31	3.2441	2.8346	-.0893	-4.9236
32	3.2787	2.7775	-.2141	-5.0999

AVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS = 2.3162

slabs, cylinders, and spheres in Eq. (38). The level moments are listed for each direction set.

The computer code used to determine these generalized direction sets is listed in Appendix A. It was also used for all applications of critical calculations involving the area method or the AD sets. The other direction sets were loaded to the full machine accuracy in the case of the P_n and $DP_{(n-2)/2}$ or the accuracy available if less was quoted.

VI. RESULTS AND COMPARISONS

In order to examine the behavior of the constructed area method direction sets, we calculated a number of simple test problems. In these computations we utilized the LSN,[17] Diamond Difference (DD), and Step (SF) Function solution methods for the Area Method (AM), the asymptotic direction (AD), P_{n-1} and $DP_{(n-2)/2}$, and the EQ_n quadrature sets.

First, we investigate critical thickness predictions and convergence as a function of n for the one-group homogeneous slab, cylinder, and spherical geometries.[3] Second, using the various direction sets, we calculate and compare k_{eff} for the Lady Godiva Benchmark problem,[14] based on Lady Godiva[15] using the Hansen-Roach six-group cross sections.[16] Approximately 2084 criticality calculations are reported and compared: 1824 one-group slab, cylinders, spheres, and 260 comparisons on the Lady Godiva benchmark calculation.

VI.A One-Group Homogeneous Slabs, Cylinders, and Spheres

We consider, first, homogeneous problems in one-space dimensional slab, cylinder, and spherical geometries with a single uniform material in one neutron energy group. All dimensions are expressed in mean-free path units ($1/\sigma$). The critical dimensions of such systems are calculated parametrically as a function of c, the excess number of neutrons per collision,

$$c = (\nu\sigma_f + \sigma_s)/\sigma_t \quad , \tag{40}$$

where ν is the number of neutrons per fission, σ_f is the macroscopic fission cross section, σ_s is the scattering cross section, and σ_t is the total cross section. The Mark boundary condition, no incoming neutron flux at the outside boundary, was used exclusively. The Marshak boundary condition, no incoming neutron current at the outside boundary, is not generally available in the S_n transport codes.

In general, as c increases, the critical dimensions decrease, and, hence, are more strongly affected by the outside boundary. Thus, while for small c-1, diffusion theory might be an accurate approximation, for higher c values, one would expect to use higher values of n in the S_n approximation for an accurate critical dimension determination. The values listed as "exact" were taken from the work of Carlson and Bell[18,19] and are based on extrapolated end-point and variational calculations.

Three numerical methods were utilized in the representation of the angular flux at the cell center in terms of the values at the boundary. These methods were LSN, Diamond Difference (DD), and the Step Function (SF) approximation.

In Table 9 we give a summary and index to the 1824 homogeneous critical calculations. Only the Area Method AM and Asymptotic Direction AD were available for cylindrical calculations. The P_{n-1} and $DP_{(n-2)/2}$ quadrature sets do not generalize to one-dimensional cylinders or multidimensional systems. The Carlson EQ_n, Case A and B, are generalizable to cylinders but were not run as such. In Tables 10-47 we summarize the one-group homogeneous critical radius calculations. The percentage differences of these critical radii and the values of Carlson and Bell[18,19] are displayed graphically in Figs. 10-47. More accurate evaluations of the integral transport equation eigenvalues one-group Benchmark problems are reported by Kaper, Lindeman, and Leaf[20] but do not significantly alter any of the conclusions herein. In particular, the "exact" value listed for slabs, C = 2.0, should be 0.3110 instead of 0.3108.[18,20] All S_2 calculations default to $\mu_1^2 = 1/3$ for all direction sets.

TABLE 9

TABLE AND FIGURE INDEX SUMMARY

ONE-GROUP HOMOGENEOUS

CRITICALITY CALCULATIONS

Method	Directions	Slab	Cylinder	Sphere
LSN	AM*	10	11	12
DD	AM	13	14	15
SF	AM	16	17	18
LSN	AD**	19	20	21
DD	AD	22	23	24
SF	AD	25	26	27
LSN	PN***	28		29
DD	PN	30		31
SF	PN	32		33
LSN	DPN#	34		35
DD	DPN	36		37
SF	DPN	38		39
LSN	CA##	40		41
DD	CA	42		43
LSN	CB###	44		45
DD	CB	46		47

* : Area Method Directions

** : Asymptotic Directions

*** : Full Range Gause-Legrendre Directions

\# : Half Range Gauss-Legrendre Directions

\## : Carlson, EQ_n, Case A

\### : Carlson, EQ_n, Case B

VI.A.1. One-Group, One-Medium Comparisons

We use the term "essentially identical" in the sense that by overlaying two graphs the differences are essentially not distinguishable by a curve width. We reference figure numbers in parentheses; e.g., LSN (10) means the LSN method, Fig. 10.

TABLE 10

HOMOGENEOUS SLABS, LSN, AREA METHOD DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	5.85572 3.35751	5.68096 .27288	5.67409 .15162	5.67020 .08296	5.66642 .01624	5.66498 -.00918	5.66550
1.05	3.49616 5.93182	3.31654 .49512	3.30998 .29635	3.30600 .17575	3.30226 .06242	3.30067 .01424	3.30020
1.10	2.31352 9.46910	2.13216 .88767	2.12416 .50913	2.11968 .29715	2.11548 .09842	2.11371 .01467	2.11340
1.20	1.48755 15.37656	1.31588 2.06158	1.30416 1.15256	1.29818 .68875	1.29278 .26991	1.29051 .09385	1.28930
1.40	.92034 24.94461	.77582 5.32487	.75969 3.13454	.75133 1.99946	.74348 .93443	.74006 .47013	.73660
1.60	.68031 32.87227	.55817 9.01680	.54112 5.68672	.53179 3.86484	.52244 2.03867	.51806 1.18262	.51200
1.80	.54341 39.80139	.43824 12.74376	.42167 8.48161	.41225 6.05814	.40231 3.50013	.39734 2.22279	.38870
2.00	.45381 46.01480	.36165 16.36229	.34602 11.33140	.33689 8.39575	.32691 5.18275	.32168 3.50161	.31080

TABLE 11

HOMOGENEOUS CYLINDER, AREA METHOD DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	8.97286 -.77892	9.06308 .21873	9.04834 .05573	9.04353 .00254	9.04006 -.03583	9.03932 -.04401	9.04330
1.05	5.36747 -.81914	5.41129 -.00942	5.41015 -.03049	5.40950 -.04250	5.40821 -.06634	5.40890 -.05359	5.41180
1.10	3.55502 -.65059	3.56759 -.29930	3.57152 -.18948	3.57257 -.16013	3.57338 -.13750	3.57381 -.12548	3.57830
1.20	2.28644 -.08565	2.27326 -.66160	2.27929 -.39809	2.28110 -.31900	2.28256 -.25520	2.28325 -.22505	2.28840
1.40	1.41389 1.18729	1.38391 -.95828	1.38919 -.58041	1.39090 -.45803	1.39236 -.35354	1.39309 -.30130	1.39730
1.60	1.04447 2.30875	1.01044 -1.02459	1.01429 -.64747	1.01553 -.52601	1.01664 -.41728	1.01725 -.35753	1.02090
1.80	.83384 3.36470	.79956 -.88534	.80221 -.55634	.80299 -.46027	.80372 -.36891	.80418 -.31288	.80670
2.00	.69606 4.30961	.66285 -.66747	.66461 -.40297	.66501 -.34317	.66541 -.28293	.66571 -.23797	.66730

TABLE 12

HOMOGENEOUS SPHERE, AREA METHOD DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	11.86690	12.06620	12.03280	12.02550	12.02100	12.02060	12.02700
	-1.33117	.32593	.04822	-.01247	-.04989	-.05321	
1.05	7.12611	7.28415	7.27301	7.27187	7.27203	7.27262	7.27720
	-2.07621	.09550	-.05758	-.07324	-.07104	-.06294	
1.10	4.73204	4.86214	4.86216	4.86404	4.86643	4.86776	4.87270
	-2.88670	-.21672	-.21631	-.17772	-.12868	-.10138	
1.20	3.04925	3.15229	3.15838	3.16181	3.16531	3.16705	3.17200
	-3.86980	-.62137	-.42938	-.32125	-.21091	-.15605	
1.40	1.88746	1.96424	1.97190	1.97560	1.97919	1.98090	1.98540
	-4.93301	-1.06578	-.67996	-.49360	-.31278	-.22665	
1.60	1.39461	1.45648	1.46392	1.46739	1.47069	1.47225	1.47610
	-5.52063	-1.32918	-.82515	-.59007	-.36651	-.26082	
1.80	1.11341	1.16542	1.17236	1.17552	1.17850	1.17991	1.18330
	-5.90636	-1.51103	-.92453	-.65748	-.40565	-.28649	
2.00	.92941	.97439	.98076	.98363	.98634	.98760	.99060
	-6.17706	-1.63598	-.99314	-.70351	-.43035	-.30305	

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TABLE 13

HOMOGENEOUS SLABS, DD, AREA METHOD DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	5.85572	5.68096	5.67409	5.67020	5.66642	5.66498	5.66550
	3.35751	.27288	.15162	.08296	.01624	-.00918	
1.05	3.49616	3.31654	3.30998	3.30600	3.30226	3.30067	3.30020
	5.93782	.49512	.29635	.17575	.06242	.01424	
1.10	2.31352	2.13216	2.12416	2.11968	2.11548	2.11371	2.11340
	9.46910	.88767	.50913	.29715	.09842	.01467	
1.20	1.48755	1.31588	1.30416	1.29818	1.29278	1.29051	1.28930
	15.37656	2.06158	1.15256	.68875	.26991	.09385	
1.40	.92034	.77598	.75969	.75134	.74348	.74006	.73660
	24.94461	5.34605	3.13454	2.00054	.93443	.47013	
1.60	.68031	.55784	.54112	.53179	.52244	.51806	.51200
	32.87227	8.95215	5.68789	3.86543	2.03926	1.18281	
1.80	.54341	.43768	.42150	.41216	.40229	.39734	.38870
	39.80139	12.60046	8.43710	6.03499	3.49550	2.22176	
2.00	.45381	.36098	.34575	.33673	.32685	.32166	.31080
	46.01480	16.14479	11.24614	8.34299	5.16474	3.49421	

TABLE 14

HOMOGENEOUS CYLINDER, DD, AREA METHOD DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	8.98655 -.62754	9.02610 -.19020	9.03493 -.09255	9.03714 -.06812	9.03747 -.06447	9.03799 -.05872	9.04330
1.05	5.37596 -.66226	5.39235 -.35940	5.40349 -.15355	5.40621 -.10329	5.40691 -.09036	5.40823 -.06597	5.41180
1.10	3.56095 -.48487	3.55650 -.60923	3.56751 -.30154	3.57055 -.21658	3.57257 -.16013	3.57338 -.13750	3.57830
1.20	2.29035 .08521	2.26651 -.95656	2.27671 -.51084	2.27978 -.37668	2.28203 -.27836	2.28297 -.23728	2.28840
1.40	1.41631 1.36048	1.37994 -1.24240	1.38757 -.69634	1.39004 -.51957	1.39200 -.37930	1.39289 -.31561	1.39730
1.60	1.04625 2.48310	1.00765 -1.29787	1.01309 -.76501	1.01487 -.59066	1.01636 -.44471	1.01709 -.37320	1.02090
1.80	.83526 3.54010	.79743 -1.14875	.80125 -.67547	.80244 -.52758	.80349 -.39841	.80404 -.32949	.80670
2.00	.69723 4.48584	.66116 -.92043	.66382 -.52210	.66455 -.41181	.66520 -.31425	.66559 -.25581	.66730

TABLE 15

HOMOGENEOUS SPHERE, DD, AREA METHOD DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	11.91320	12.01520	12.01450	12.01700	12.01770	12.01900	12.02700
	-.94620	-.09811	-.10393	-.08315	-.07733	-.06652	
1.05	7.15751	7.25841	7.26422	7.26772	7.27047	7.27185	7.27720
	-1.64473	-.25820	-.17837	-.13027	-.09248	-.07352	
1.10	4.75407	4.84802	4.85739	4.86177	4.86561	4.86736	4.87270
	-2.43458	-.50650	-.31420	-.22431	-.14550	-.10959	
1.20	3.06377	3.14471	3.15575	3.16057	3.16486	3.16682	3.17200
	-3.41204	-.86034	-.51230	-.36034	-.22509	-.16330	
1.40	1.89645	1.95977	1.97035	1.97487	1.97893	1.98077	1.98540
	-4.48021	-1.29092	-.75803	-.53037	-.32588	-.23320	
1.60	1.40122	1.45336	1.46283	1.46688	1.47051	1.47216	1.47610
	-5.07283	-1.54055	-.89899	-.62462	-.37870	-.26692	
1.80	1.11866	1.16304	1.17151	1.17512	1.17836	1.17984	1.18330
	-5.46269	-1.71216	-.99637	-.69129	-.41748	-.29240	
2.00	.93377	.97243	.98005	.98330	.98622	.98754	.99060
	-5.73723	-1.83394	-1.06461	-.73713	-.44236	-.30890	

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TABLE 16

HOMOGENEOUS SLABS, SF, AREA METHOD DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	7.55855 33.41356	6.41102 13.15902	6.14959 8.54461	6.02215 6.29511	5.89853 4.11311	5.83806 3.04577	5.66550
1.05	4.00366 21.31577	3.53590 7.14205	3.45294 4.62815	3.41183 3.38259	3.37185 2.17119	3.35244 1.58286	3.30020
1.10	2.48894 17.76932	2.20865 4.50690	2.17372 2.85436	2.15621 2.02543	2.13937 1.22889	2.13144 .85363	2.11340
1.20	1.52973 18.64775	1.33532 3.56927	1.31655 2.11326	1.30717 1.38636	1.29851 .71454	1.29471 .41924	1.28930
1.40	.91645 24.41580	.77552 5.28434	.75952 3.11200	.75115 1.97504	.74328 .90627	.73985 .44113	.73660
1.60	.66795 30.45972	.55407 8.21741	.53860 5.19550	.52996 3.50776	.52123 1.80193	.51713 1.00180	.51200
1.80	.52919 36.14282	.43318 11.44395	.41856 7.68226	.41002 5.48433	.40087 3.13180	.39628 1.94924	.38870
2.00	.43954 41.42289	.35645 14.68837	.34282 10.30104	.33461 7.65957	.32546 4.71676	.32062 3.16011	.31080

TABLE 17

HOMOGENEOUS CYLINDER, SF, AREA METHOD DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	13.82588 52.88537	10.93942 20.96715	10.24504 13.28875	9.91293 9.61625	9.59995 6.15538	9.45023 4.49976	9.04330
1.05	6.94112 28.25903	5.95953 10.12112	5.74775 6.20765	5.64822 4.36860	5.55552 2.65573	5.51235 1.85792	5.41180
1.10	4.18145 16.85586	3.73984 4.51445	3.66540 2.43423	3.63182 1.49577	3.60000 .60643	3.59017 .33174	3.57830
1.20	2.50717 9.55976	2.29601 .33265	2.27712 -.49278	2.27016 -.79713	2.26648 -.95796	2.26653 -.95580	2.28840
1.40	1.47357 5.45804	1.36022 -2.65389	1.35965 -2.69477	1.36123 -2.58136	1.36537 -2.28505	1.36819 -2.08331	1.39730
1.60	1.06451 4.27141	.98126 -3.88273	.98341 -3.67241	.98600 -3.41854	.99077 -2.95122	.99448 -2.58746	1.02090
1.80	.83886 3.98715	.77106 -4.41745	.77344 -4.12342	.77588 -3.81992	.78022 -3.28238	.78357 -2.86761	.80670
2.00	.69424 4.03754	.63628 -4.64896	.63835 -4.33832	.64044 -4.02512	.64418 -3.46511	.64710 -3.02762	.66730

TABLE 18

HOMOGENEOUS SPHERE, SF, AREA METHOD DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	20.84789 73.34240	15.53263 29.14802	14.24624 18.45212	13.61457 13.20004	13.04729 8.48331	12.76421 6.12962	12.02700
1.05	10.07936 38.50602	8.33189 14.49401	7.91290 8.73555	7.72466 6.14881	7.55010 3.75011	7.46920 2.63838	7.27720
1.10	5.93391 21.77874	5.19829 6.68194	5.05179 3.67542	4.98641 2.33352	4.92939 1.16338	4.90518 .66663	4.87270
1.20	3.49403 10.15242	3.20339 .98975	3.16503 -.21970	3.15107 -.65975	3.14300 -.91418	3.14220 -.93959	3.17200
1.40	2.02547 2.01799	1.91974 -3.30725	1.92104 -3.24143	1.92539 -3.02239	1.93403 -2.58758	1.94077 -2.24789	1.98540
1.60	1.45404 -1.49474	1.39832 -5.26908	1.40750 -4.64715	1.41509 -4.13347	1.42618 -3.38183	1.43371 -2.87156	1.47610
1.80	1.14156 -3.52782	1.10708 -6.44108	1.11826 -5.49663	1.12639 -4.80919	1.13753 -3.86803	1.14479 -3.25453	1.18330
2.00	.94238 -4.86785	.91899 -7.22846	.93047 -6.07019	.93843 -5.26639	.94902 -4.19723	.95579 -3.51363	.99060

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TABLE 19

HOMOGENEOUS SLABS, LSN, ASYMPTOTIC DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	5.85572	5.66895	5.65520	5.65262	5.65262	5.65373	5.66550
	3.35752	.06083	-.18181	-.22742	-.22729	-.20777	
1.05	3.49616	3.30351	3.28908	3.28662	3.28699	3.28832	3.30020
	5.93784	.10043	-.33709	-.41137	-.40024	-.36002	
1.10	2.31352	2.11854	2.10219	2.09913	2.09931	2.10000	2.11340
	9.46896	.24312	-.53050	-.67499	-.66657	-.63435	
1.20	1.48755	1.30255	1.28142	1.27657	1.27553	1.27654	1.28930
	15.37630	1.02803	-.61147	-.98766	-1.06806	-.98965	
1.40	.97034	.76444	.73832	.72994	.72558	.72525	.73660
	24.94460	3.78009	.23365	-.90447	-1.49670	-1.54022	
1.60	.68031	.54859	.52209	.51196	.50496	.50319	.51200
	32.87224	7.14678	1.97062	-.00850	-1.37483	-1.72148	
1.80	.54341	.43005	.40486	.39425	.38579	.38290	.38870
	39.80140	10.63862	4.15701	1.42776	-.74853	-1.49214	
2.00	.45381	.35453	.33109	.32062	.31153	.30793	.31080
	46.01474	14.07151	6.52751	3.16013	.23648	-.92264	

TABLE 20

HOMOGENEOUS CYLINDER, LSN, AREA METHOD DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	8.97286	9.05301	9.03366	9.02991	9.02934	9.03040	9.04330
	-.77888	.10734	-.10657	-.14805	-.15442	-.14265	
1.05	5.36747	5.40213	5.39562	5.39585	5.39792	5.39988	5.41180
	-.81911	-.17871	-.29900	-.29470	-.25645	-.22027	
1.10	3.55502	3.55915	3.55830	3.56015	3.56345	3.56575	3.57830
	-.65048	-.53505	-.55902	-.50724	-.41503	-.35084	
1.20	2.28644	2.26520	2.26658	2.26922	2.27312	2.27562	2.28840
	-.08560	-1.01380	-.95331	-.83833	-.66779	-.55852	
1.40	1.41389	1.37658	1.37727	1.37970	1.38346	1.38589	1.39730
	1.18699	-1.48262	-1.43361	-1.25950	-.99034	-.81677	
1.60	1.04447	1.00375	1.00310	1.00488	1.00813	1.01034	1.02090
	2.30869	-1.67986	-1.74402	-1.56891	-1.25135	-1.03466	
1.80	.83384	.79346	.79171	.79288	.79554	.79751	.80670
	3.36475	-1.64161	-1.85824	-1.71371	-1.38378	-1.13979	
2.00	.69606	.65727	.65477	.65541	.65754	.65926	.66730
	4.30956	-1.50382	-1.87774	-1.78173	-1.46287	-1.20538	

TABLE 21

HOMOGENEOUS SPHERE, LSN, ASYMPTOTIC DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	11.86691	12.05862	12.02241	12.01571	12.01320	12.01389	12.02700
	-1.33105	.26287	-.03815	-.09386	-.11472	-.10900	
1.05	7.12611	7.27939	7.26553	7.26442	7.26572	7.26735	7.27720
	-2.07618	.03007	-.16031	-.17559	-.15772	-.13532	
1.10	4.73204	4.85880	4.85658	4.85844	4.86160	4.86370	4.87270
	-2.88667	-.28517	-.33072	-.29260	-.22779	-.18473	
1.20	3.04925	3.14973	3.15441	3.15791	3.16201	3.16428	3.17200
	-3.86979	-.70214	-.55466	-.44412	-.31492	-.24337	
1.40	1.88746	1.96271	1.96947	1.97321	1.97721	1.97927	1.98540
	-4.93323	-1.14309	-.80244	-.61373	-.41263	-.30868	
1.60	1.39461	1.45536	1.46219	1.46571	1.46931	1.47111	1.47610
	-5.52044	-1.40493	-.94206	-.70369	-.46023	-.33772	
1.80	1.11341	1.16454	1.17101	1.17423	1.17745	1.17905	1.18330
	-5.90597	-1.58556	-1.03872	-.76682	-.49405	-.35891	
2.00	.92941	.97362	.97964	.98258	.98549	.98692	.99060
	-6.17703	-1.71399	-1.10591	-.81005	-.51545	-.37151	

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TABLE 22

HOMOGENEOUS SLABS, DD, ASYMPTOTIC DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	5.85572 3.35752	5.66895 .06083	5.65520 -.18181	5.65262 -.22742	5.65262 -.22729	5.65373 -.20777	5.66550
1.05	3.49616 5.93784	3.30351 .1[illegible]043	3.28908 -.33709	3.28662 -.41137	3.28699 -.40024	3.28832 -.36002	3.30020
1.10	2.31352 9.46896	2.11884 .24312	2.10219 -.53050	2.09913 -.67499	2.09931 -.66657	2.10000 -.63405	2.11340
1.20	1.48755 15.37630	1.30255 1.02803	1.28142 -.61147	1.27657 -.98766	1.27553 -1.068[illegible]6	1.27654 -.98965	1.28930
1.40	.92034 24.94460	.76462 3.8[illegible]389	.73832 .23365	.72995 -.90339	.72558 -1.49670	.72525 -1.54022	.73660
1.60	.68031 32.87224	.54825 7.08062	.52207 1.96736	.51196 -.00860	.5[illegible]497 -1.37394	.50319 -1.72130	.51200
1.80	.54341 39.80140	.42939 10.46911	.40465 4.10399	.39415 1.40133	.38577 -.75354	.38290 -1.49302	.38870
2.00	.45381 46.01474	.35387 13.85795	.33080 6.43436	.32046 3.10665	.31148 .21824	.30791 -.92983	.31080

TABLE 23

HOMOGENEOUS CYLINDER, DD, ASYMPTOTIC DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	8.98655 -.62757	9.01762 -.29057	9.02121 -.24424	9.02424 -.21078	9.02705 -.17964	9.02923 -.15555	9.04330
1.05	5.37596 -.66229	5.38377 -.51793	5.38959 -.41036	5.39292 -.34892	5.39678 -.27750	5.39929 -.23112	5.41180
1.10	3.56095 -.48485	3.54838 -.83628	3.55459 -.66267	3.55835 -.55747	3.56277 -.43398	3.56540 -.36057	3.57830
1.20	2.29035 .08531	2.25869 -1.29808	2.26476 -1.05505	2.26807 -.88849	2.27268 -.68775	2.27539 -.56858	2.28840
1.40	1.41631 1.36047	1.37272 -1.75922	1.37580 -1.53894	1.37894 -1.31365	1.38316 -1.01190	1.38573 -.82825	1.39730
1.60	1.04625 2.48303	1.00162 -1.94722	1.00198 -1.85334	1.00429 -1.62682	1.00788 -1.27521	1.01021 -1.04754	1.02090
1.80	.83526 3.54004	.79137 -1.89972	.79081 -1.96982	.79238 -1.77473	.79533 -1.40977	.79739 -1.15398	.80670
2.00	.69723 4.48588	.65561 -1.75236	.65402 -1.99024	.65499 -1.84493	.65735 -1.49060	.65915 -1.22078	.66730

TABLE 24

HOMOGENEOUS SPHERES, DD, ASYMPTOTIC DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	11.91316 -.94654	12.00974 -.1434[illegible]	12.00627 -.17236	12.00854 -.15348	12.01048 -.13737	12.01255 -.12011	12.02700
1.05	7.15751 -1.64473	7.25427 -.31509	7.25751 -.27059	7.26066 -.22731	7.26440 -.17591	7.26672 -.14408	7.27720
1.10	4.75407 -2.43456	4.845[illegible]1 -.56822	4.85223 -.42020	4.85643 -.33397	4.86091 -.24192	4.86337 -.19139	4.87270
1.20	3.[illegible]6377 -3.41220	3.14266 -.92487	3.15219 -.62447	3.15692 -.47543	3.16169 -.32516	3.16413 -.24802	3.17200
1.40	1.89645 -4.48031	1.95848 -1.35584	1.96817 -.86807	1.97266 -.64170	1.97703 -.42145	1.97919 -.31262	1.98540
1.60	1.40122 -5.07296	1.45240 -1.6[illegible]591	1.46127 -1.00501	1.46532 -.73058	1.46918 -.46867	1.47106 -.34147	1.47610
1.80	1.11866 -5.46267	1.16229 -1.77581	1.17029 -1.09943	1.17392 -.79297	1.17736 -.50218	1.17901 -.36250	1.18330
2.00	.93377 -5.73722	.97179 -1.89880	.97906 -1.16487	.98233 -.83529	.98542 -.52333	.98689 -.37498	.99060

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TABLE 25

HOMOGENEOUS SLABS, SF, ASYMPTOTIC DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	7.55885 33.41356	6.39048 12.79643	6.11986 8.01978	5.99646 5.84167	5.87980 3.78251	5.82372 2.79261	5.66550
1.05	4.00366 21.31577	3.51990 6.65707	3.42835 3.88320	3.38959 2.70865	3.35493 1.65838	3.33914 1.17978	3.30020
1.10	2.48894 17.76932	2.19387 3.80767	2.15002 1.73274	2.13429 .98869	2.12236 .42405	2.11786 .21091	2.11340
1.20	1.52973 18.64775	1.32173 2.51535	1.29326 .30713	1.28503 -.33114	1.28089 -.65213	1.28047 -.68502	1.28930
1.40	.91645 24.41580	.76423 3.75118	.73821 .21823	.72975 -.93007	.72530 -1.53343	.72498 -1.57777	.73660
1.60	.66795 30.45972	.54464 6.37520	.51976 1.51572	.51026 -.34013	.50380 -1.60185	.50228 -1.89917	.51200
1.80	.52919 36.14282	.42515 9.37703	.40196 3.41258	.39219 .89780	.38445 -1.09243	.38189 -1.75199	.38870
2.00	.43954 41.42289	.34948 12.44375	.32810 5.56540	.31851 2.48099	.31020 -.19391	.30694 -1.24157	.31080

TABLE 26

HOMOGENEOUS CYLINDER, SF, ASYMPTOTIC DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	13.82588	10.90750	10.20418	9.87978	9.57848	9.43495	9.04330
	52.88537	20.61415	12.83691	9.24976	5.91797	4.33080	
1.05	6.94112	5.94178	5.72308	5.62743	5.54095	5.50153	5.41180
	28.25903	9.79301	5.75189	3.98438	2.38651	1.65800	
1.10	4.18145	3.72663	3.64607	3.61490	3.59008	3.58069	3.57830
	16.85586	4.14523	1.89387	1.02275	.32926	.06666	
1.20	2.50717	2.28532	2.26072	2.25540	2.25540	2.25787	2.28840
	9.55976	-.13459	-1.20955	-1.44211	-1.44223	-1.33423	
1.40	1.47357	1.35145	1.34549	1.34813	1.35525	1.36097	1.39730
	5.45804	-3.28120	-3.70814	-3.51912	-3.00908	-2.59974	
1.60	1.06451	.97369	.97064	.97394	.98126	.98688	1.02090
	4.27141	-4.62466	-4.92293	-4.60015	-3.88248	-3.33261	
1.80	.83886	.76435	.76177	.76467	.77121	.77629	.80670
	3.98715	-5.24975	-5.56944	-5.21053	-4.39898	-3.76959	
2.00	.69424	.63025	.62762	.62996	.63562	.64012	.66730
	4.03754	-5.55181	-5.94699	-5.59583	-4.74815	-4.07351	

TABLE 27

HOMOGENEOUS SPHERES, SF, ASYMPTOTIC DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	20.84789 73.34240	15.48356 28.74001	14.18823 17.96984	13.56913 12.82224	13.02128 8.26710	12.74643 5.98182	12.02700
1.05	10.07936 38.50602	8.29559 13.99425	7.88456 8.34604	7.70221 5.84033	7.53594 3.55548	7.45931 2.50252	7.27720
1.10	5.93391 21.77874	5.18461 6.40124	5.03379 3.30588	4.97183 2.03447	4.91983 .96713	4.89804 .52003	4.87270
1.20	3.49403 10.15242	3.19440 .70614	3.15295 -.60057	3.14105 -.97558	3.13628 -1.12603	3.13730 -1.09387	3.17200
1.40	2.02547 2.01799	1.91385 -3.60371	1.91285 -3.65394	1.91852 -3.36868	1.92936 -2.82237	1.93744 -2.41577	1.98540
1.60	1.45404 -1.49474	1.39377 -5.57748	1.40108 -5.08207	1.40968 -4.49997	1.42251 -3.63043	1.43106 -3.05140	1.47610
1.80	1.14156 -3.52782	1.10333 -6.75857	1.11292 -5.94739	1.12189 -5.19007	1.13448 -4.12597	1.14259 -3.44042	1.18330
2.00	.94238 -4.86785	.91578 -7.55251	.92588 -6.53314	.93455 -5.65809	.94640 -4.46195	.95391 -3.70389	.99060

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TABLE 28

HOMOGENEOUS SLABS, LSN, P_n DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	5.85572 3.35752	5.68534 .35021	5.67268 .12679	5.66890 .06009	5.66628 .01371	5.66556 .00108	5.66550
1.05	3.49616 5.93784	3.32110 .63344	3.30832 .24613	3.30461 .13376	3.30221 .06089	3.30141 .03669	3.30020
1.10	2.31352 9.46896	2.13639 1.08780	2.12148 .38230	2.11763 .19991	2.11514 .08246	2.11434 .04430	2.11340
1.20	1.48755 15.37630	1.31908 2.30946	1.29882 .73826	1.29409 .37145	1.29132 .15671	1.29043 .08803	1.28930
1.40	.92034 24.94460	.77755 5.55997	.75025 1.85294	.74281 .84268	.73894 .31738	.73786 .17059	.73660
1.60	.68031 32.87224	.55917 9.21233	.52997 3.50909	.52029 1.61948	.51483 .55238	.51344 .28042	.51200
1.80	.54341 39.80140	.43885 12.90236	.41018 5.52672	.39932 2.73202	.39236 .94103	.39051 .46647	.38870
2.00	.45381 46.01474	.36205 16.48956	.33480 7.72136	.32351 4.09040	.31545 1.49767	.31311 .74273	.31080

TABLE 29

HOMOGENEOUS SPHERES, LSN, P_n DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	11.86691 -1.33105	12.06494 .31546	12.02996 .02459	12.02376 -.02691	12.02108 -.04924	12.02170 -.04406	12.02700
1.05	7.12611 -2.07618	7.28473 .10350	7.27400 -.04403	7.27319 -.05507	7.27413 -.04215	7.27493 -.03116	7.27720
1.10	4.73204 -2.88667	4.86330 -.19284	4.86466 -.16503	4.86682 -.12063	4.86937 -.06839	4.87059 -.04326	4.87270
1.20	3.04925 -3.86979	3.15349 -.58357	3.16125 -.33902	3.16507 -.21844	3.16850 -.11039	3.16992 -.06564	3.17200
1.40	1.88746 -4.93323	1.96537 -1.00909	1.97441 -.55343	1.97856 -.34461	1.98202 -.17016	1.98338 -.10168	1.98540
1.60	1.39461 -5.52044	1.45747 -1.26183	1.46621 -.67020	1.46999 -.41362	1.47314 -.20026	1.47437 -.11737	1.47610
1.80	1.11341 -5.90597	1.16631 -1.43604	1.17439 -.75257	1.17782 -.46326	1.18065 -.22359	1.18175 -.13096	1.18330
2.00	.92941 -6.17703	.97516 -1.55824	.98258 -.80996	.98568 -.49644	.98824 -.23795	.98923 -.13871	.99060

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TABLE 30

HOMOGENEOUS SLABS, DD, P_n DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	5.85572 3.35752	5.68534 .35021	5.67568 .12679	5.66890 .06009	5.66628 .01371	5.66556 .00108	5.66550
1.05	3.49616 5.93784	3.32110 .63344	3.30832 .24613	3.30461 .13376	3.30221 .06089	3.30141 .03669	3.30020
1.10	2.31352 9.46896	2.13639 1.08780	2.12148 .38230	2.11763 .19991	2.11514 .08246	2.11434 .04430	2.11340
1.20	1.48755 15.37630	1.31908 2.30946	1.29882 .73826	1.29409 .37145	1.29132 .15671	1.29043 .08803	1.28930
1.40	.92034 24.94460	.77773 5.58324	.75025 1.85294	.74281 .84268	.73894 .31738	.73786 .17059	.73660
1.60	.68031 32.87224	.55883 9.14646	.52099 3.01463	.52032 1.62462	.51483 .55257	.51344 .28042	.51200
1.80	.54341 39.80140	.43827 12.75324	.40095 5.46730	.39928 2.72147	.39236 .94039	.39051 .46627	.38870
2.00	.45381 46.31474	.36134 16.26205	.33444 7.60696	.32336 4.04183	.31544 1.49225	.31310 .74122	.31080

TABLE 31

HOMOGENEOUS SPHERES, DD, P_n DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	11.91316 -.94654	12.01689 -.08404	12.01856 -.07013	12.02027 -.05596	12.02077 -.05178	12.02187 -.04264	12.02700
1.05	7.15751 -1.64473	7.26822 -.23326	7.26958 -.11850	7.27161 -.07678	7.27411 -.04252	7.27511 -.02866	7.27720
1.10	4.75407 -2.43456	4.84993 -.46736	4.86780 -.22363	4.86610 -.13543	4.86945 -.06660	4.87076 -.03976	4.87270
1.20	3.06377 -3.41220	3.14645 -.80533	3.15691 -.38121	3.16482 -.22641	3.16862 -.10656	3.17006 -.06129	3.17200
1.40	1.89645 -4.48631	1.96121 -1.21821	1.97383 -.58282	1.97852 -.34674	1.98213 -.16472	1.98348 -.09677	1.98540
1.60	1.40122 -5.07296	1.45457 -1.45842	1.46579 -.69814	1.47000 -.41303	1.47323 -.19414	1.47444 -.11227	1.47610
1.80	1.11866 -5.46267	1.16409 -1.62345	1.17408 -.77920	1.17784 -.46106	1.18073 -.21710	1.18181 -.12576	1.18330
2.00	.93377 -5.73722	.97336 -1.74020	.98532 -.83553	.98571 -.49393	.98831 -.23122	.98928 -.13346	.99060

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TABLE 32

HOMOGENEOUS SLABS, SF, P_n DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	7.55855 33.41356	6.41647 13.25520	6.14189 8.40854	6.01558 6.17907	5.89539 4.05768	5.83683 3.02418	5.66550
1.05	4.00366 21.31577	3.54067 7.28662	3.44857 4.49565	3.40816 3.27120	3.37048 2.12947	3.35237 1.58073	3.30020
1.10	2.48894 17.76932	2.21285 4.70551	2.16954 2.65638	2.15286 1.86708	2.13828 1.17708	2.13160 .86099	2.11340
1.20	1.52973 18.64775	1.33844 3.81104	1.31045 1.64022	1.30233 1.01081	1.29658 .56454	1.29433 .39000	1.28930
1.40	.91645 24.41580	.77722 5.51490	.74990 1.80605	.74233 .77752	.73845 .25115	.73745 .11483	.73660
1.60	.66795 30.45972	.55506 8.41081	.52751 3.02981	.51842 1.25298	.51347 .28740	.51238 .07327	.51200
1.80	.52919 36.14282	.43380 11.60188	.40723 4.76648	.39717 2.18005	.39089 .56316	.38937 .17364	.38870
2.00	.43954 41.42289	.35685 14.81606	.33178 6.75012	.32137 3.40181	.31404 1.04350	.31203 .39468	.31080

TABLE 33

HOMOGENEOUS SPHERES, SF, P_n DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	20.84789 73.34240	15.54670 29.26502	14.23471 18.35632	13.61152 13.17473	13.05935 8.58363	12.78248 6.28155	12.02700
1.05	10.07936 38.50602	8.34197 14.63156	7.91609 8.77933	7.73490 6.28947	7.56876 4.00643	7.49097 2.93747	7.27720
1.10	5.93301 21.77874	5.23073 7.34762	5.05964 3.83657	5.00010 2.61456	4.94921 1.57019	4.92695 1.11343	4.87270
1.20	3.49403 10.15242	3.20991 1.19509	3.17411 .06650	3.16501 -.22049	3.16154 -.32972	3.16195 -.31673	3.17200
1.40	2.02547 2.01799	1.92464 -3.06018	1.92927 -2.82736	1.93752 -2.41159	1.94942 -1.81218	1.95687 -1.43685	1.98540
1.60	1.45404 -1.49474	1.40231 -4.99895	1.41467 -4.16178	1.42548 -3.42955	1.43923 -2.49785	1.44725 -1.95458	1.47610
1.80	1.14156 -3.52782	1.11045 -6.15609	1.12456 -4.96442	1.13548 -4.04148	1.14884 -2.91242	1.15646 -2.26810	1.18330
2.00	.94238 -4.86785	.92192 -6.93297	.93607 -5.50450	.94649 -4.45312	.95900 -3.19036	.96605 -2.47789	.99060

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TABLE 34

HOMOGENEOUS SLABS, LSN, DP_n DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	5.85572 3.35752	5.66741 .03377	5.68460 .33714	5.67087 .09482	5.66564 .00241	5.66494 -.00986	5.66550
1.05	3.49616 5.93784	3.29911 -.03317	3.30114 .02859	3.30084 .01928	3.30097 .02347	3.30000 -.00606	3.30020
1.10	2.31352 9.46896	2.10748 -.28008	2.11388 .02274	2.11355 .00719	2.11346 .00268	2.11340 .00008	2.11340
1.20	1.48755 15.37630	1.27755 -.91109	1.28979 .03771	1.28947 .01345	1.28942 .00959	1.28940 .00783	1.28930
1.40	.92034 24.94460	.72319 -1.82048	.73575 -.11530	.73678 .02404	.73661 .00180	.73661 .00178	.73660
1.60	.68031 32.87274	.50290 -1.77735	.50963 -.46352	.51202 .00386	.51197 -.00631	.51197 -.00587	.51200
1.80	.54341 39.80140	.38495 -.96526	.38561 -.79510	.38848 -.05657	.38880 .02544	.38877 .01866	.38870
2.00	.45381 46.01474	.31175 .30428	.30775 -.97998	.31033 -.15204	.31108 .08895	.31102 .07190	.31080

TABLE 35

HOMOGENEOUS SPHERES, LSN, DP_n DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	11.86691 -1.33105	11.99667 -.25221	12.01238 -.12157	12.01678 -.08500	12.01932 -.06384	12.02113 -.04880	12.02700
1.05	7.12611 -2.07618	7.24849 -.39448	7.26540 -.16221	7.27022 -.09588	7.27353 -.05049	7.27483 -.03255	7.27720
1.10	4.73204 -2.88667	4.84437 -.58147	4.86108 -.23848	4.86598 -.13788	4.86947 -.06622	4.87080 -.03896	4.87270
1.20	3.04925 -3.86979	3.14551 -.83523	3.16097 -.34769	3.16567 -.19959	3.16906 -.09269	3.17032 -.05291	3.17200
1.40	1.88746 -4.93323	1.96278 -1.13938	1.97568 -.48936	1.97976 -.28412	1.98273 -.13450	1.98383 -.07924	1.98540
1.60	1.39461 -5.52044	1.45670 -1.31430	1.46769 -.56965	1.47123 -.32977	1.47382 -.15455	1.47478 -.08956	1.47610
1.80	1.11341 -5.90597	1.16632 -1.43515	1.17587 -.62760	1.17899 -.36425	1.18127 -.17141	1.18212 -.09969	1.18330
2.00	.92941 -6.17703	.97554 -1.51993	.98400 -.66618	.98676 -.38734	.98880 -.18125	.98956 -.10482	.99060

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TABLE 36

HOMOGENEOUS SLABS, DD, DP_n DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	5.85572 3.35752	5.66741 .03377	5.66582 .00568	5.66536 -.00249	5.66469 -.01435	5.66469 -.01431	5.66550
1.05	3.49616 5.93784	3.29911 -.03317	3.30114 .02859	3.30084 .01928	3.30000 -.00606	3.30000 -.00606	3.30020
1.10	2.31352 9.46896	2.10748 -.28008	2.11388 .02274	2.11355 .00719	2.11346 .00268	2.11340 .00008	2.11340
1.20	1.48755 15.37630	1.27755 -.91109	1.28979 .03771	1.28947 .01345	1.28942 .00959	1.28940 .00783	1.28930
1.40	.92034 24.94460	.72319 -1.82048	.73575 -.11530	.73678 .02404	.73661 .00180	.73661 .00178	.73660
1.60	.68031 32.87224	.50271 -1.81425	.50963 -.46352	.51202 .00386	.51197 -.00631	.51197 -.00587	.51200
1.80	.54341 39.80140	.38401 -1.20775	.38563 -.79016	.38848 -.05720	.38880 .02544	.38877 .01866	.38870
2.00	.45381 46.01474	.31081 .00430	.30768 -1.00410	.31032 -.15536	.31108 .08873	.31102 .07172	.31080

TABLE 37

HOMOGENEOUS SPHERES, DD, DP_n DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	11.91316	11.98227	12.01420	12.01907	12.02061	12.02188	12.02700
	-.94654	-.37190	-.10641	-.06597	-.05310	-.04259	
1.05	7.15751	7.24158	7.26709	7.27190	7.27451	7.27540	7.27720
	-1.64473	-.48952	-.13887	-.07277	-.03692	-.02472	
1.10	4.75407	4.84102	4.86263	4.86740	4.87022	4.87123	4.87270
	-2.43456	-.65006	-.20657	-.10884	-.05081	-.03020	
1.20	3.06377	3.14426	3.16225	3.16672	3.16959	3.17062	3.17200
	-3.41220	-.87447	-.30726	-.16658	-.07604	-.04363	
1.40	1.89645	1.96253	1.97662	1.98046	1.98307	1.98401	1.98540
	-4.48031	-1.15207	-.44225	-.24860	-.11736	-.06989	
1.60	1.40122	1.45671	1.46843	1.47177	1.47407	1.47491	1.47610
	-5.07296	-1.31370	-.51978	-.29346	-.13761	-.08030	
1.80	1.11866	1.16642	1.17648	1.17942	1.18147	1.18223	1.18330
	-5.46267	-1.42666	-.57624	-.32761	-.15433	-.09052	
2.00	.93377	.97568	.98451	.98713	.98897	.98965	.99060
	-5.73722	-1.50611	-.61519	-.35069	-.16425	-.09578	

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TABLE 38

HOMOGENEOUS SLABS, SF, DP_n DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	7.55855 33.41356	6.35059 12.09232	6.12014 8.02471	6.00544 6.00010	5.89182 3.99471	5.83510 2.99358	5.66550
1.05	4.00366 21.31577	3.49945 6.03744	3.43521 4.09094	3.40161 3.07291	3.36800 2.05456	3.35110 1.54241	3.30020
1.10	2.48894 17.76932	2.17409 2.87176	2.15856 2.13672	2.14730 1.60382	2.13610 1.07406	2.13045 .80683	2.11340
1.20	1.52973 18.64775	1.29276 .26873	1.29939 .78261	1.29684 .58507	1.29441 .39633	1.29317 .30022	1.28930
1.40	.91645 24.41580	.72247 -1.91775	.73431 -.31051	.73573 -.11822	.73595 -.08843	.73612 -.06458	.73660
1.60	.66795 30.45972	.49957 -2.42752	.50670 -1.03478	.50972 -.44518	.51046 -.29985	.51085 -.22557	.51200
1.80	.52919 36.14282	.38101 -1.97835	.38263 -1.56195	.38607 -.67724	.38720 -.38540	.38758 -.28912	.38870
2.00	.43954 41.42289	.30773 -.98622	.30499 -1.86915	.30807 -.87984	.30956 -.40022	.30989 -.29404	.31080

TABLE 39

HOMOGENEOUS SPHERES, SF, DP_n DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	20.84789 73.34240	15.36240 27.73257	14.20310 18.09346	13.61132 13.17300	13.06990 8.67128	12.79282 6.36755	12.02700
1.05	10.07936 38.50602	8.27654 13.73250	7.92342 8.88001	7.75066 6.50613	7.58364 4.21100	7.50304 3.10345	7.27720
1.10	5.93391 21.77874	5.19516 6.61773	5.07738 4.20049	5.01926 3.00772	4.96429 1.87967	4.93866 1.35368	4.87270
1.20	3.49403 10.15242	3.21547 1.37031	3.19413 .69779	3.18342 .35993	3.17501 .09480	3.17219 .00588	3.17200
1.40	2.02547 2.01799	1.93505 -2.53617	1.94696 -1.93636	1.95261 -1.65131	1.96005 -1.27671	1.96483 -1.03583	1.98540
1.60	1.45404 -1.49474	1.41262 -4.30019	1.42989 -3.13087	1.43818 -2.56897	1.44798 -1.90474	1.45379 -1.51171	1.47610
1.80	1.14156 -3.52782	1.12000 -5.34911	1.13782 -3.84336	1.14639 -3.11889	1.15629 -2.28281	1.16199 -1.80127	1.18330
2.00	.94238 -4.86785	.93066 -6.05111	.94781 -4.31982	.95606 -3.48679	.96548 -2.53559	.97084 -1.99425	.99060

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TABLE 40

HOMOGENEOUS SLABS, LSN, CA DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	5.85572 3.35752	5.67404 .15069	5.66650 .01768	5.66554 .00062	5.66486 -.01135	5.66479 -.01248	5.66550
1.05	3.49616 5.93784	3.30688 .29230	3.30157 .04156	3.30104 .02541	3.30000 -.00606	3.30000 -.00666	3.30020
1.10	2.31352 9.46896	2.12113 .36578	2.11471 .06207	2.11393 .02508	2.11357 .00820	2.11346 .00307	2.11340
1.20	1.48755 15.37630	1.30454 1.18215	1.29285 .27550	1.29071 .10959	1.28981 .03923	1.28959 .02262	1.28930
1.40	.92034 24.94460	.76589 3.97679	.74653 1.34742	.74076 .56465	.73782 .16532	.73718 .07871	.73660
1.60	.68031 32.87224	.54974 7.37158	.52788 3.10178	.51958 1.48138	.51430 .45000	.51300 .19599	.51200
1.80	.54341 39.80140	.43101 10.88448	.40910 5.24732	.39958 2.79959	.39248 .97136	.39040 .43758	.38870
2.00	.45381 46.01474	.35535 14.33445	.33431 7.56555	.32440 4.37472	.31613 1.71570	.31335 .82006	.31080

TABLE 41

HOMOGENEOUS SPHERES, LSN, CA DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	11.86691 -1.33105	12.06755 .33720	12.02787 .00726	12.02259 -.03664	12.02036 -.05523	12.02119 -.04830	12.02700
1.05	7.12611 -2.07618	7.28499 .10708	7.27121 -.08228	7.27139 -.07989	7.27311 -.05623	7.27427 -.04032	7.27720
1.10	4.73204 -2.88667	4.86258 -.20775	4.86186 -.22238	4.86489 -.16036	4.86827 -.09094	4.86989 -.05776	4.87270
1.20	3.04925 -3.86979	3.15218 -.62500	3.15877 -.41715	3.16328 -.27504	3.16746 -.14300	3.16925 -.08658	3.17200
1.40	1.88746 -4.93323	1.96423 -1.06632	1.97258 -.64555	1.97711 -.41749	1.98117 -.21293	1.98283 -.12919	1.98540
1.60	1.39461 -5.52044	1.45649 -1.32857	1.46470 -.77235	1.46882 -.49318	1.47243 -.24838	1.47391 -.14835	1.47610
1.80	1.11341 -5.90597	1.16544 -1.50933	1.17311 -.86112	1.17683 -.54679	1.18005 -.27506	1.18136 -.16412	1.18330
2.00	.92941 -6.17703	.97438 -1.63782	.98146 -.92298	.98482 -.58361	.98771 -.29174	.98888 -.17340	.99060

TABLE 42

HOMOGENEOUS SLABS, DD, CA DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	5.85572 3.35752	5.67404 .15069	5.66650 .01768	5.66554 .00062	5.66486 -.01135	5.66479 -.01248	5.66550
1.05	3.49616 5.93784	3.30688 .20230	3.30157 .04156	3.30104 .02541	3.30000 -.00606	3.30000 -.00606	3.30020
1.10	2.31352 9.46896	2.12113 .36578	2.11471 .06207	2.11393 .02508	2.11357 .00820	2.11346 .00307	2.11340
1.20	1.48755 15.37630	1.30454 1.18215	1.29285 .27550	1.29071 .10959	1.28981 .03923	1.28959 .02262	1.28930
1.40	.92034 24.94460	.76607 4.00048	.74653 1.34742	.74076 .56465	.73782 .16532	.73718 .07871	.73660
1.60	.68031 32.87224	.54940 7.30549	.52788 3.10235	.51961 1.48687	.51431 .45023	.51300 .19601	.51200
1.80	.54341 39.80140	.43035 10.71474	.40884 5.18048	.39951 2.78090	.39247 .97104	.39040 .43705	.38870
2.00	.45381 46.01474	.35469 14.12184	.33395 7.44714	.32422 4.31692	.31610 1.70568	.31334 .81824	.31080

TABLE 43

HOMOGENEOUS SPHERES, DD, CA DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	11.91316	12.01874	12.01446	12.01751	12.01939	12.02104	12.02700
	-.94654	-.06867	-.10429	-.07892	-.06326	-.04954	
1.05	7.15751	7.25988	7.26455	7.26907	7.27272	7.27427	7.27720
	-1.64473	-.23807	-.17378	-.11166	-.06155	-.04030	
1.10	4.75407	4.84875	4.85829	4.86368	4.86814	4.86994	4.87270
	-2.43456	-.49156	-.29582	-.18517	-.09361	-.05654	
1.20	3.06377	3.14508	3.15702	3.16274	3.16746	3.16933	3.17200
	-3.41220	-.84857	-.47214	-.29178	-.14314	-.08427	
1.40	1.89645	1.95998	1.97167	1.97692	1.98121	1.98290	1.98540
	-4.48031	-1.28013	-.69180	-.42728	-.21099	-.12608	
1.60	1.40122	1.45351	1.46405	1.46869	1.47248	1.47396	1.47610
	-5.07296	-1.53051	-.81648	-.50184	-.24552	-.14490	
1.80	1.11866	1.16316	1.17261	1.17673	1.18009	1.18140	1.18330
	-5.46267	-1.70224	-.90346	-.55494	-.27167	-.16050	
2.00	.93377	.97253	.98105	.98474	.98775	.98892	.99060
	-5.73722	-1.82378	-.96393	-.59135	-.28802	-.16967	

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TABLE 44

HOMOGENEOUS SLABS, LSN, CB DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	5.85572	5.66895	5.66800	5.66712	5.66581	5.66922	5.66550
	3.35752	.06082	.04407	.02866	.00542	.06567	
1.05	3.49616	3.30351	3.30313	3.30270	3.30167	3.30336	3.30020
	5.93784	.10042	.08886	.07584	.04458	.09584	
1.10	2.31352	2.11854	2.11623	2.11559	2.11457	2.11546	2.11340
	9.46896	.24312	.13388	.10347	.05552	.09736	
1.20	1.48755	1.30255	1.29427	1.29222	1.29078	1.29102	1.28930
	15.37630	1.02802	.38541	.22685	.11446	.13319	
1.40	.92034	.76444	.74770	.74195	.73864	.73819	.73660
	24.94460	3.78008	1.50722	.72675	.27724	.21527	
1.60	.68031	.54859	.52887	.52054	.51500	.51380	.51200
	32.87224	7.14677	3.29504	1.66894	.58616	.35126	
1.80	.54341	.43005	.40994	.40038	.39307	.39106	.38870
	39.80140	10.63861	5.46518	3.00412	1.12514	.60686	
2.00	.45381	.35453	.33505	.32507	.31665	.31391	.31080
	46.01474	14.07150	7.80276	4.59191	1.88306	1.00020	

TABLE 45

HOMOGENEOUS SPHERES, LSN, CB DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	11.86691 -1.33105	12.05861 .26287	12.02871 .01423	12.02344 -.02964	12.02091 -.05061	12.03024 .02690	12.02700
1.05	7.12611 -2.07618	7.27939 .03007	7.27179 -.07439	7.27202 -.07123	7.27353 -.05036	7.27991 .03726	7.27720
1.10	4.73204 -2.48667	4.85880 -.28517	4.86226 -.21434	4.86534 -.15100	4.86858 -.08448	4.87365 .01957	4.87270
1.20	3.04925 -3.86979	3.14973 -.70215	3.15900 -.40973	3.16356 -.26604	3.16767 -.13662	3.17171 -.00929	3.17200
1.40	1.88746 -4.93323	1.96271 -1.14309	1.97270 -.63948	1.97726 -.41004	1.98128 -.20745	1.98436 -.05218	1.98540
1.60	1.39461 -5.52044	1.45536 -1.40493	1.46477 -.76736	1.46891 -.48702	1.47250 -.24369	1.47504 -.07163	1.47610
1.80	1.11341 -5.90597	1.16454 -1.58556	1.17316 -.85692	1.17689 -.54161	1.18009 -.27099	1.18226 -.08757	1.18330
2.00	.92941 -6.17703	.97362 -1.71399	.98149 -.91938	.98486 -.57918	.98775 -.28816	.98964 -.09678	.99060

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TABLE 46

HOMOGENEOUS SLABS, DD, CB DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	5.85572 3.35752	5.66895 .06082	5.66800 .04407	5.66712 .02866	5.66581 .00542	5.62885 -.64686	5.66550
1.05	3.49616 5.93784	3.30351 .10042	3.30313 .08886	3.30270 .07584	3.30167 .04458	3.28169 -.56086	3.30020
1.10	2.31352 9.46896	2.11854 .24312	2.11623 .13388	2.11559 .10347	2.11457 .05552	2.10326 -.47971	2.11340
1.20	1.48755 15.37630	1.30255 1.02802	1.29427 .38541	1.29222 .22685	1.29078 .11446	1.28477 -.35166	1.28930
1.40	.92034 24.94460	.76462 3.80388	.74770 1.50722	.74195 .72675	.73864 .27724	.73537 -.16686	.73660
1.60	.68031 32.87224	.54825 7.08061	.52887 3.29533	.52057 1.67440	.51500 .58640	.51214 .02780	.51200
1.80	.54341 39.80140	.42939 10.46910	.40968 5.39819	.40030 2.98525	.39307 1.12457	.38995 .32149	.38870
2.00	.45381 46.01474	.35387 13.85794	.33468 7.68413	.32489 4.53387	.31662 1.87298	.31310 .74156	.31080

TABLE 47

HOMOGENEOUS SPHERES, DD, CB DIRECTIONS

C	SN=2	SN=4	SN=6	SN=8	SN=12	SN=16	EXACT ANSWER
1.02	11.91316 -.94654	12.00974 -.14349	12.01522 -.09794	12.01833 -.07208	12.01994 -.05869	11.93603 -.75635	12.02700
1.05	7.15751 -1.64473	7.25427 -.31510	7.26510 -.16629	7.26970 -.10309	7.27315 -.05571	7.22327 -.74111	7.27720
1.10	4.75407 -2.43456	4.84501 -.56822	4.85866 -.28807	4.86413 -.17586	4.86845 -.08718	4.83574 -.75841	4.87270
1.20	3.06377 -3.41220	3.14266 -.92488	3.15725 -.46494	3.16303 -.28281	3.16766 -.13678	3.14686 -.79263	3.17200
1.40	1.89645 -4.48031	1.95848 -1.35585	1.97178 -.68600	1.97706 -.41987	1.98132 -.20554	1.96860 -.84617	1.98540
1.60	1.40122 -5.07296	1.45240 -1.60592	1.46412 -.81173	1.46878 -.49571	1.47254 -.24086	1.46320 -.87413	1.47610
1.80	1.11866 -5.46267	1.16229 -1.77582	1.17266 -.89948	1.17679 -.54978	1.18013 -.26764	1.17269 -.89666	1.18330
2.00	.93377 -5.73722	.97179 -1.89880	.98109 -.96052	.98479 -.58693	.98778 -.28446	.98158 -.91096	.99060

COMPLETE V1LCRCR1ZW 2

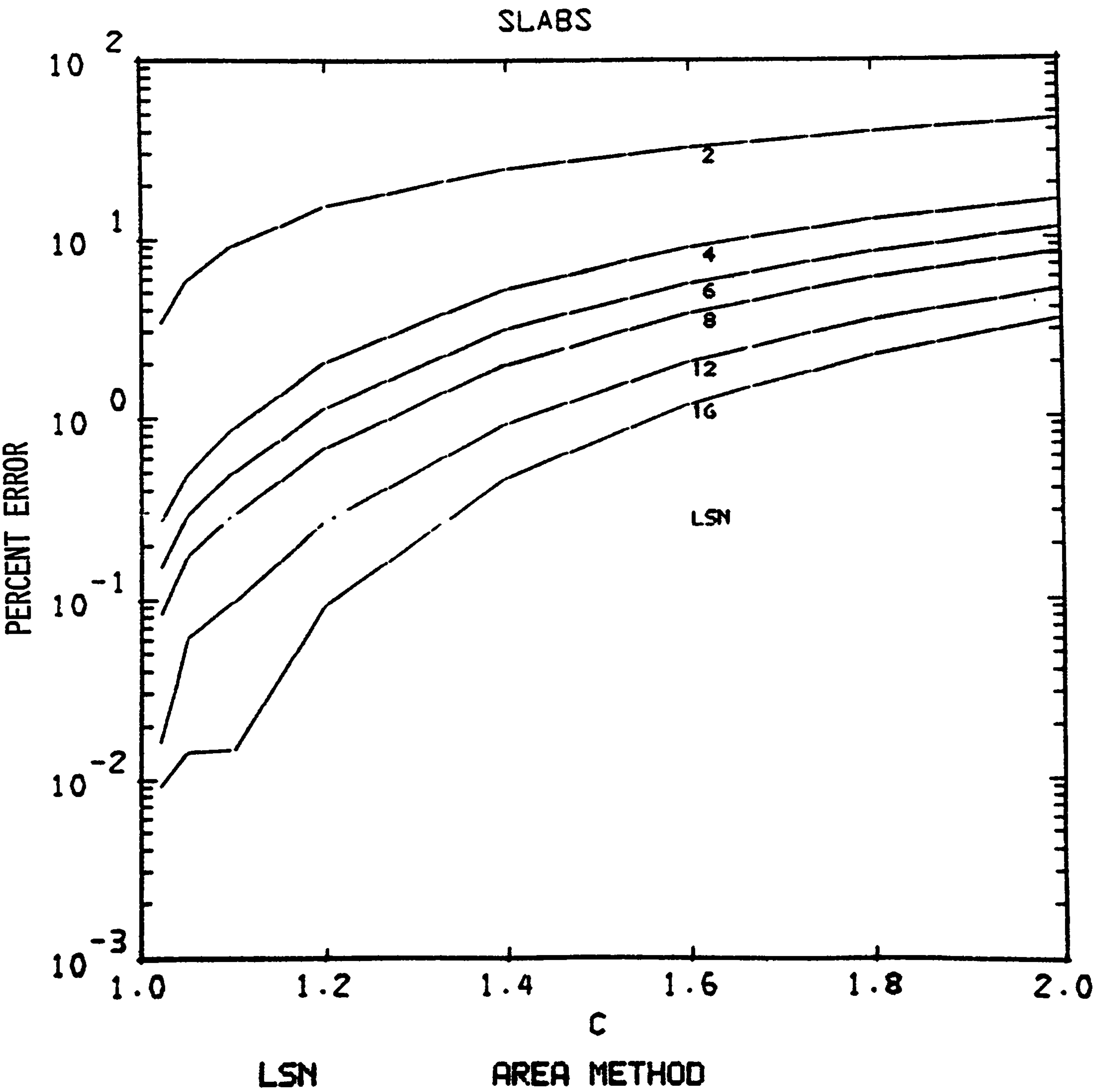

Figure 10. Percent error of critical half thickness for slabs versus c using LSN with Area Method directions.

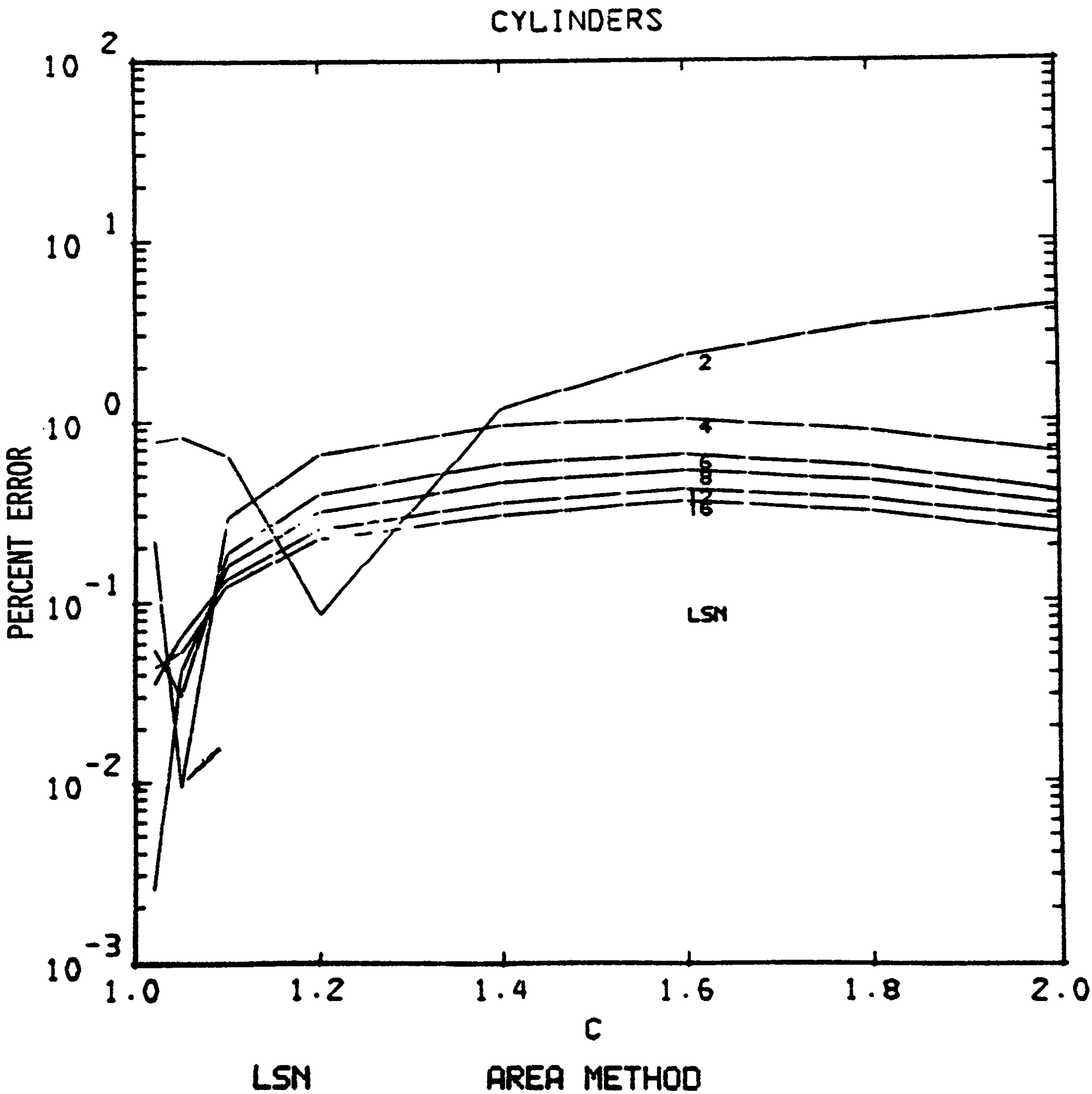

Figure 11. Percent error of critical radius for cylinders versus c using LSN with Area Method directions.

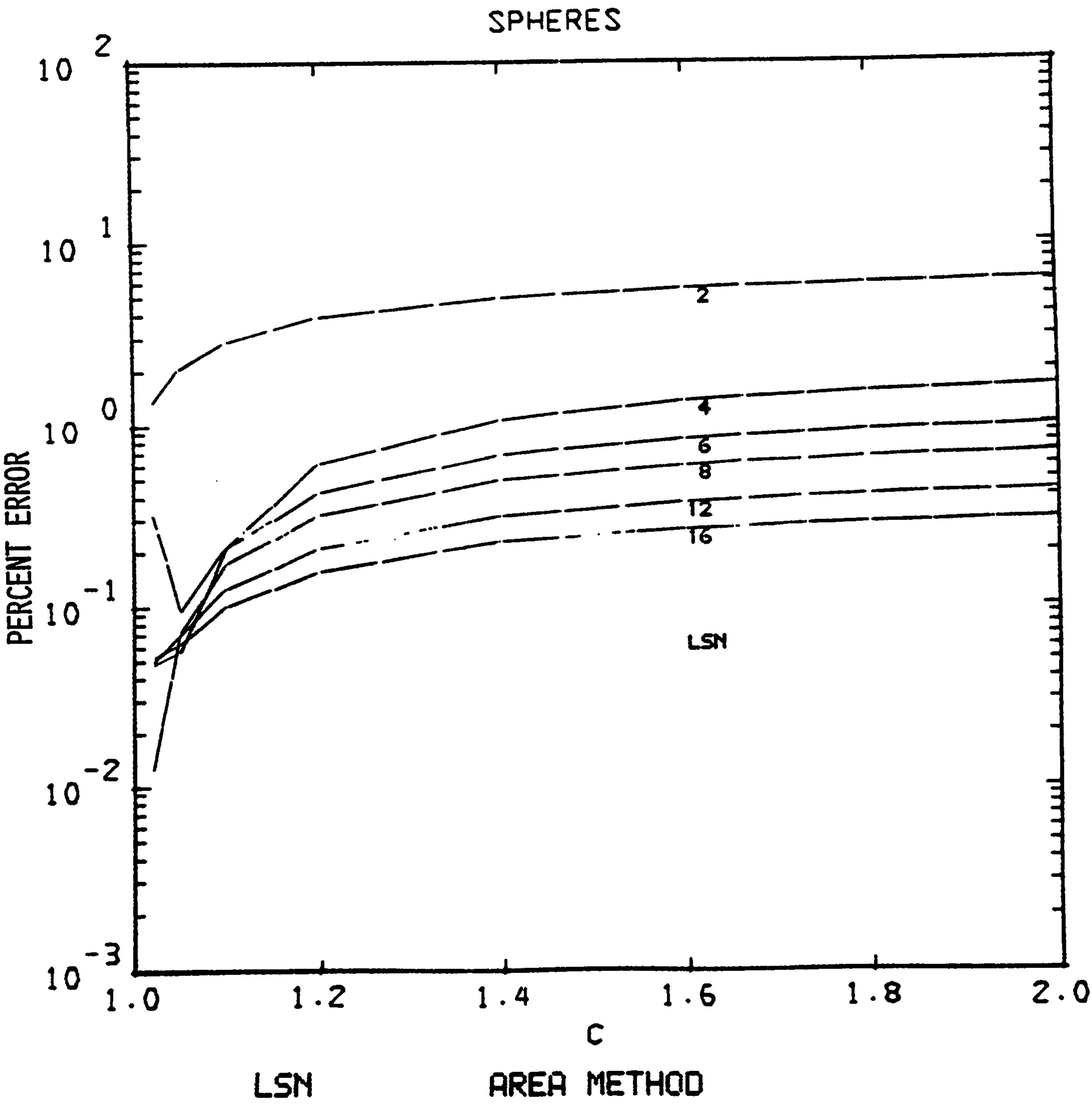

Figure 12. Percent error of critical radius for spheres versus c using LSN with Area Method directions.

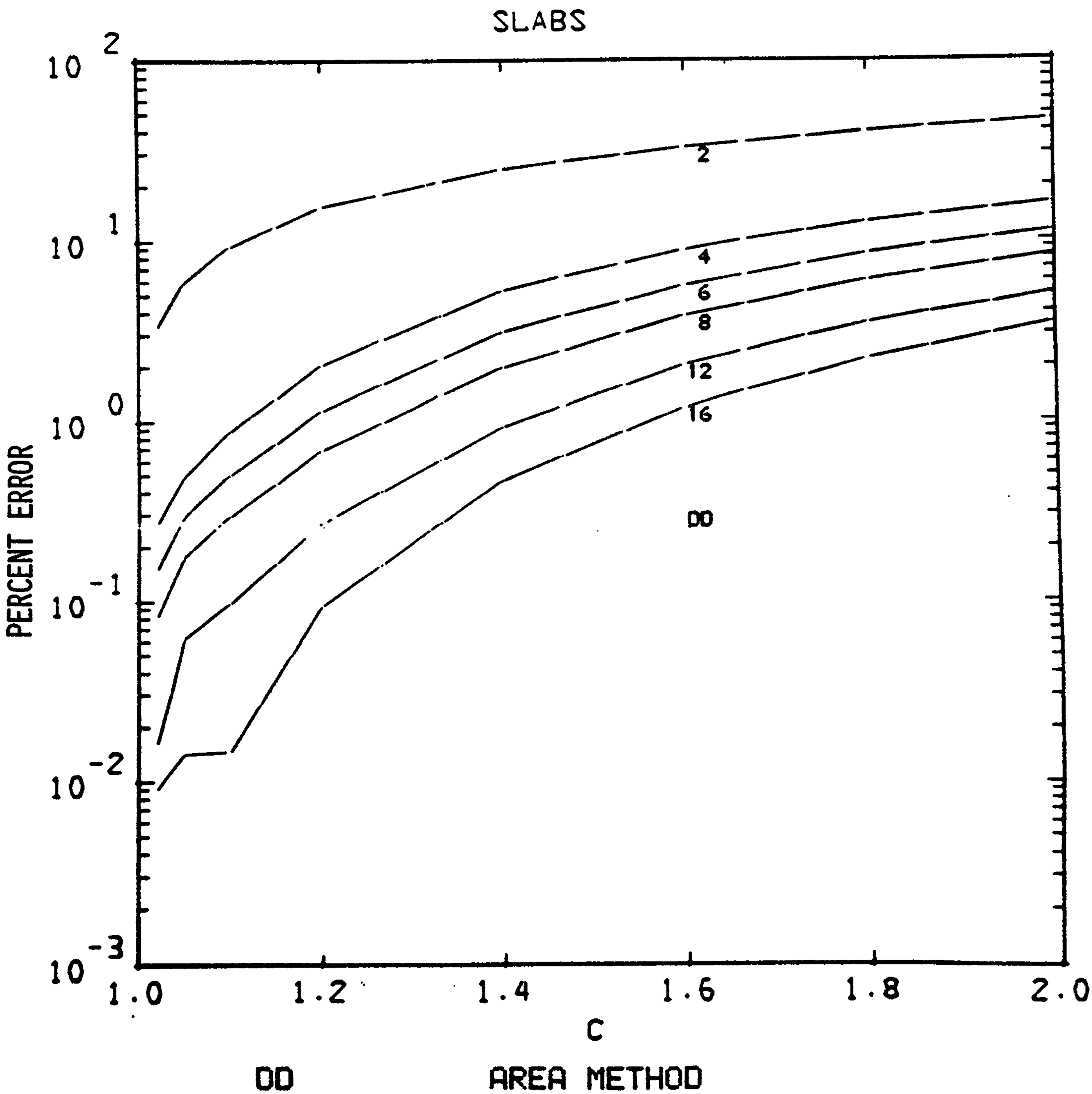

Figure 13. Percent error of critical radius for cylinders versus c using DD with Area Method directions.

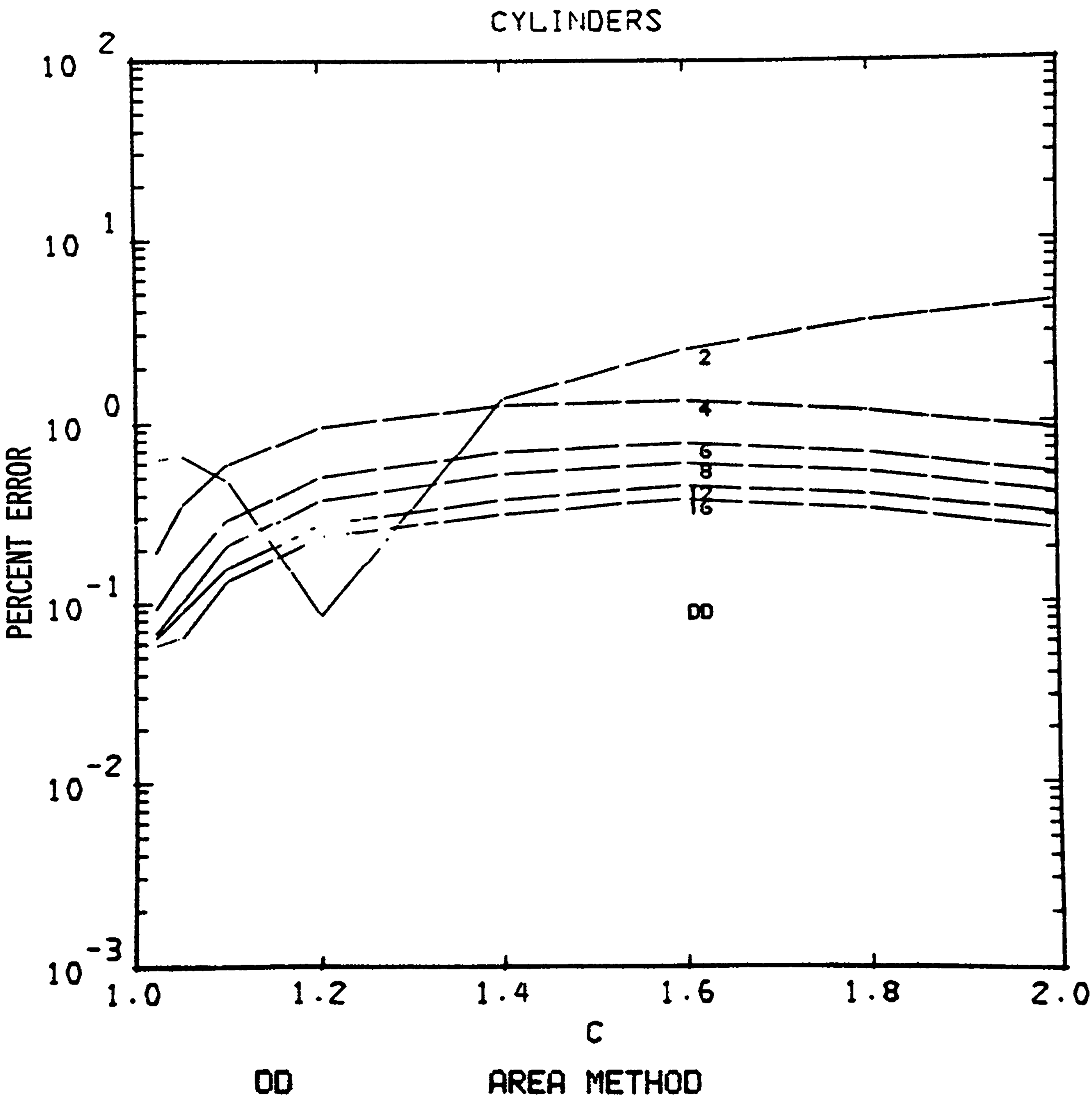

Figure 14. Percent error of critical radius for cylinders versus c using DD with Area Method directions.

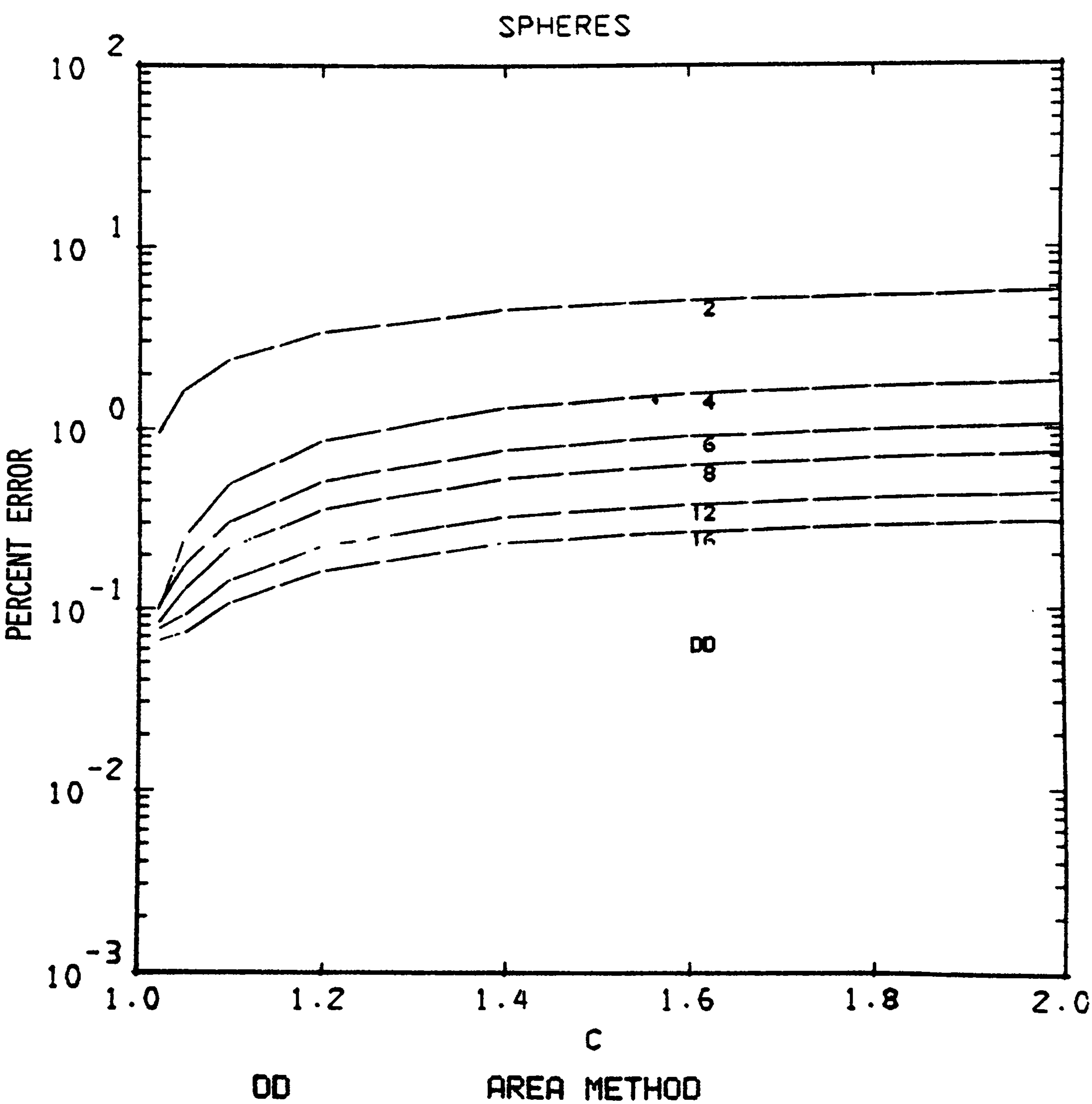

Figure 15. Percent error of critical radius for spheres versus c using DD with Area Method directions.

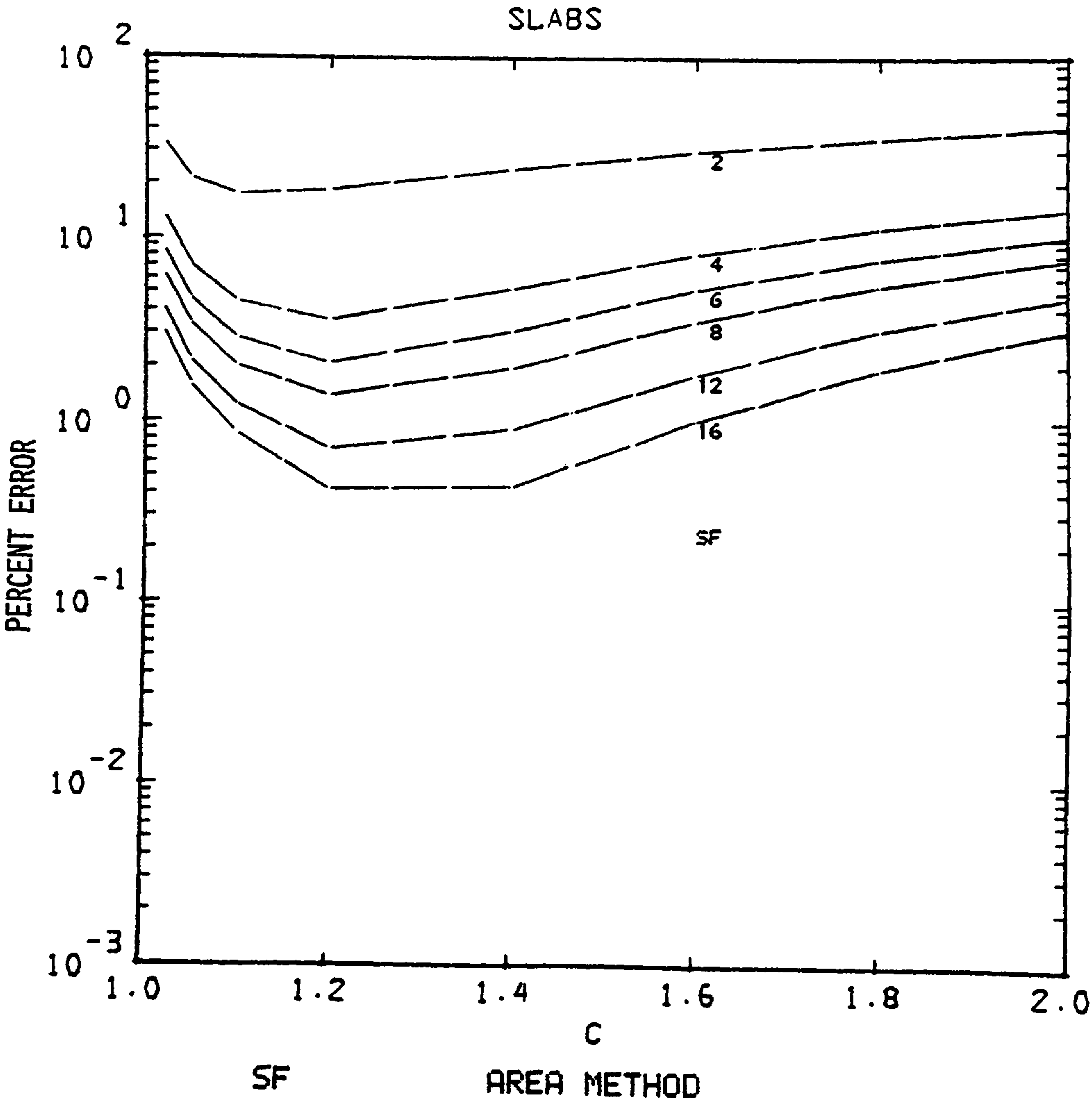

Figure 16. Percent error of critical half thickness for slabs versus c using SF with Area Method directions.

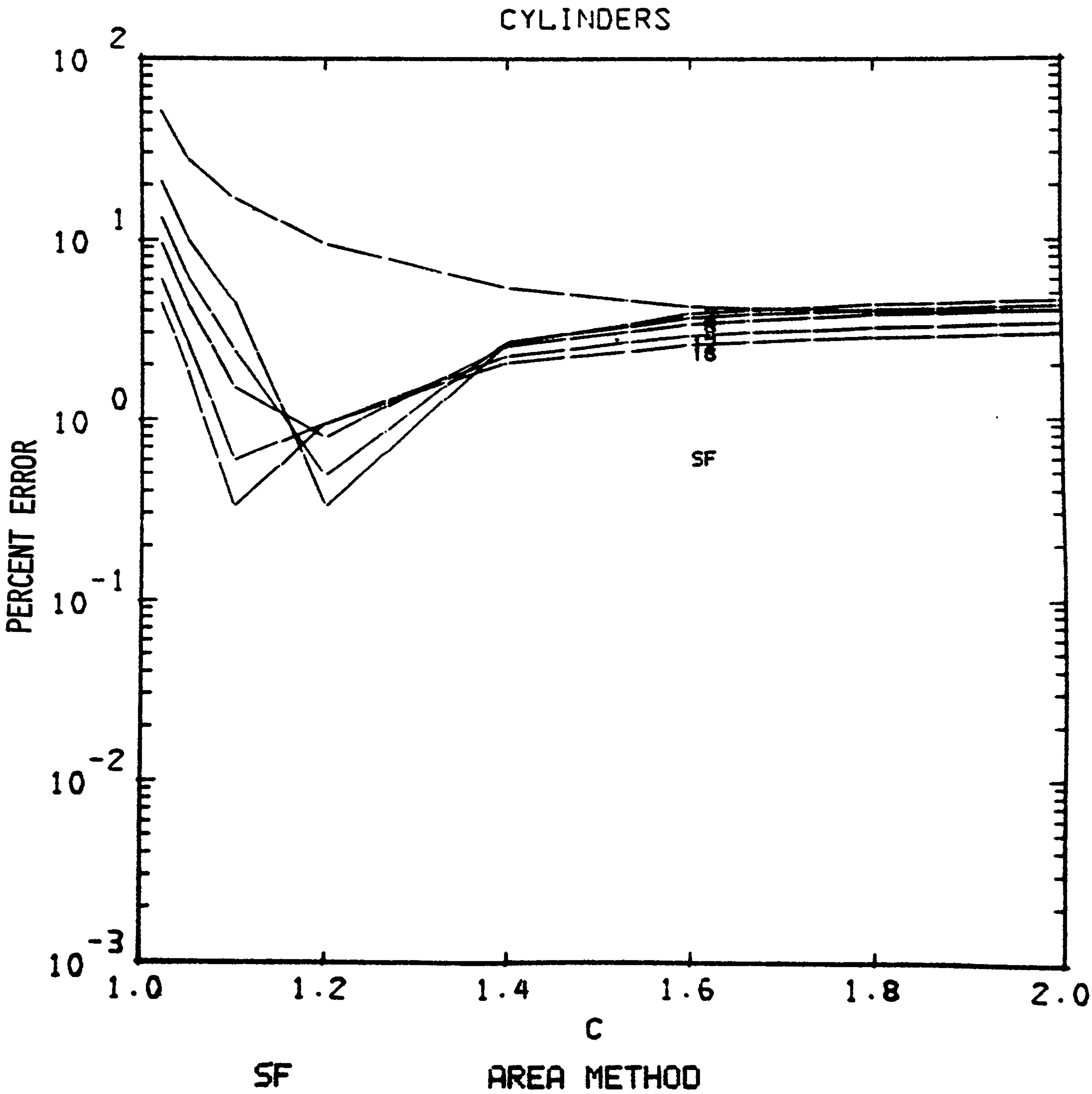

Figure 17. Percent error of critical radius for cylinders versus c using SF with Area Method directions.

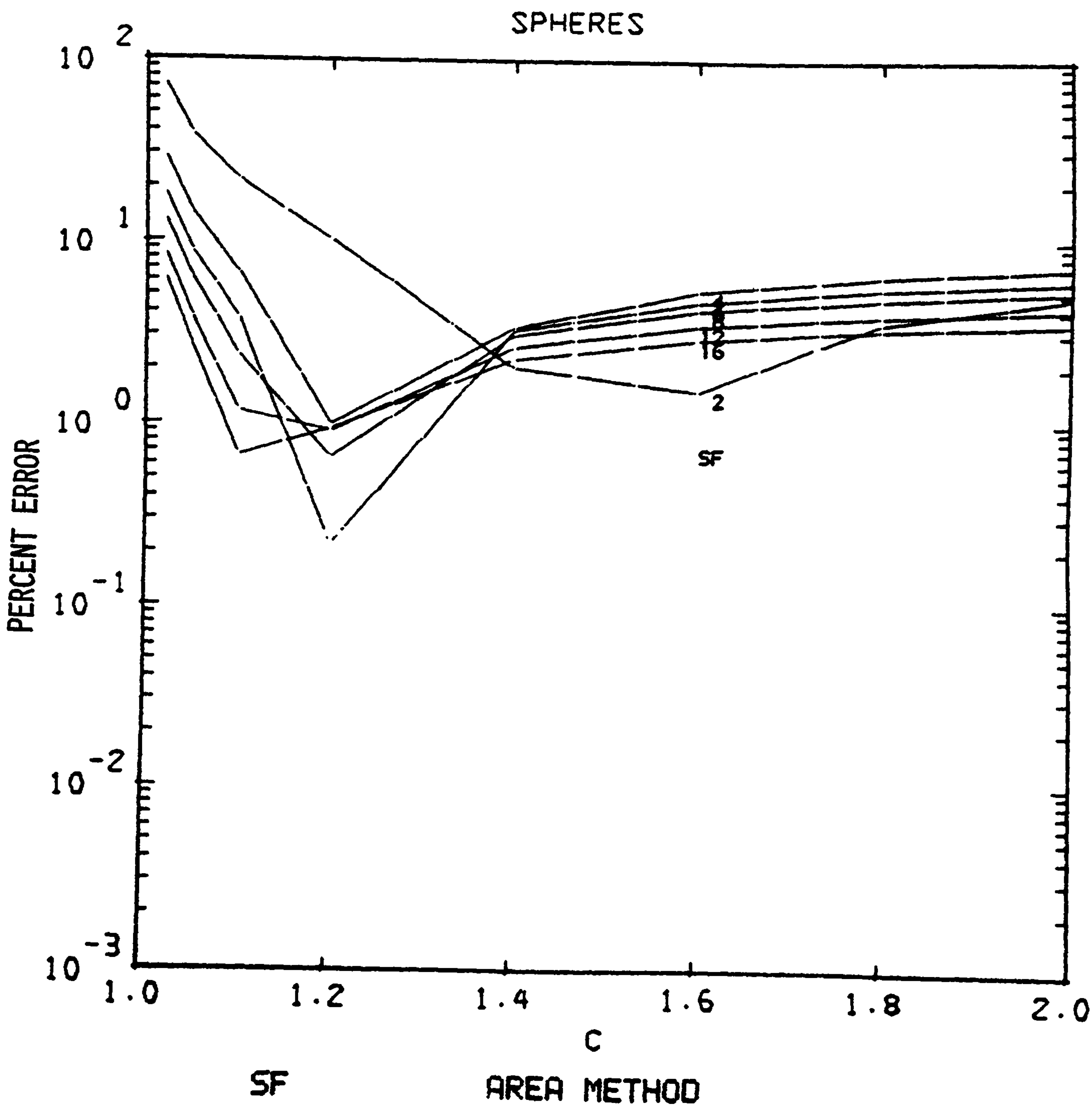

Figure 18. Percent error of critical radius for spheres versus c using SF with Area Method directions.

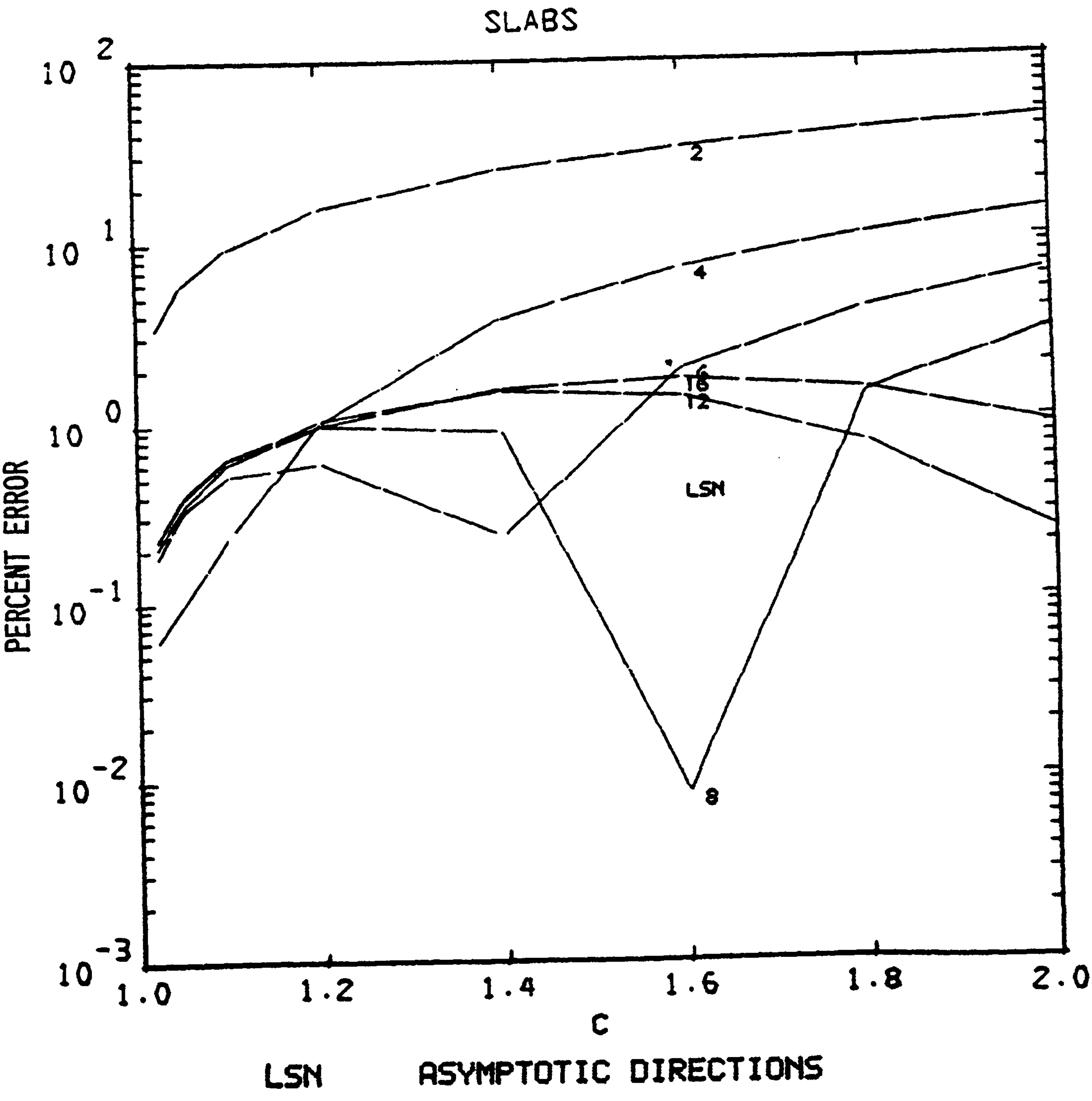

Figure 19. Percent error of critical half thickness for slabs versus c using LSN with asymptotic directions.

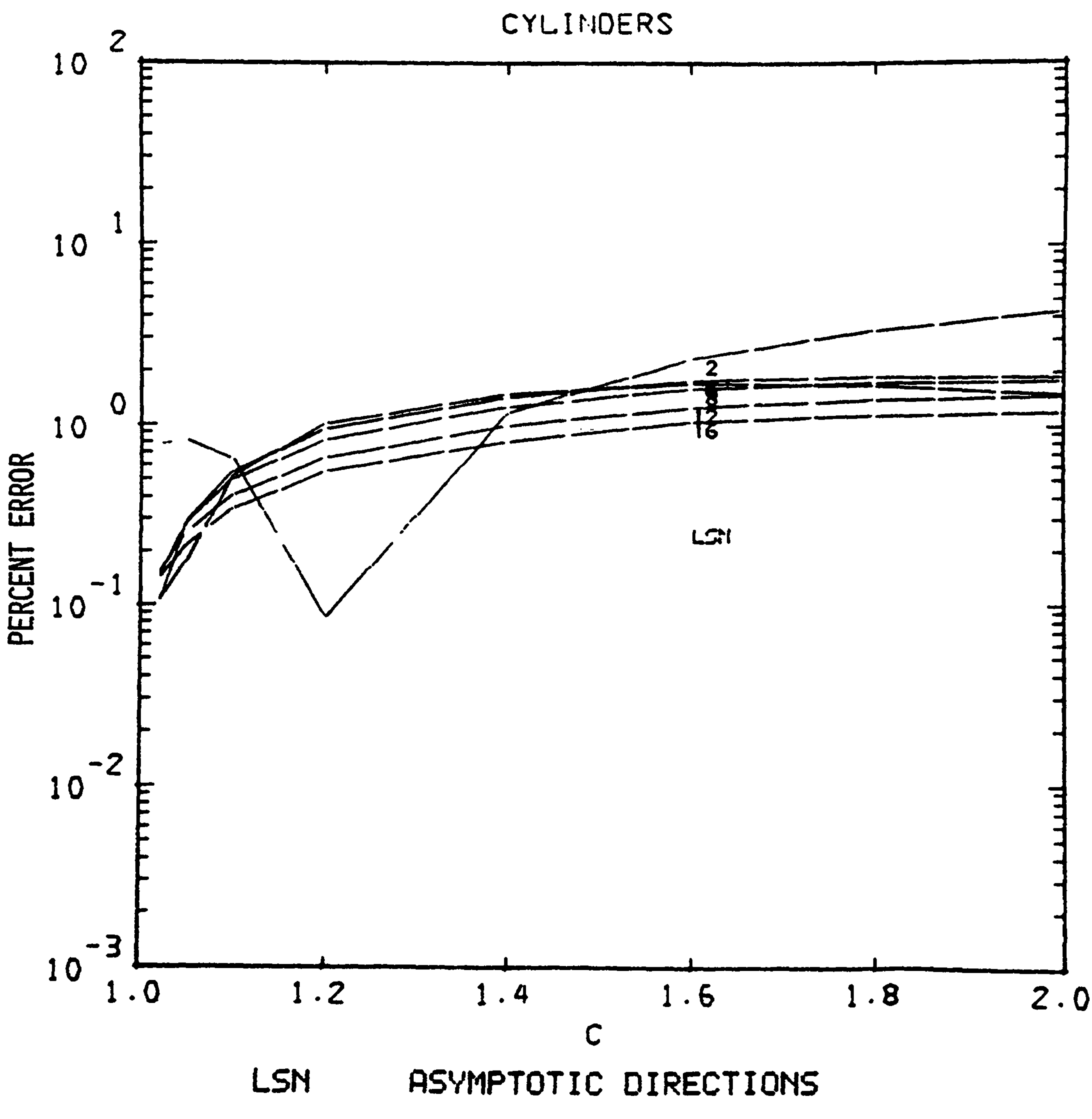

Figure 20. Percent error of critical radius for cylinders versus c using LSN with asymptotic directions.

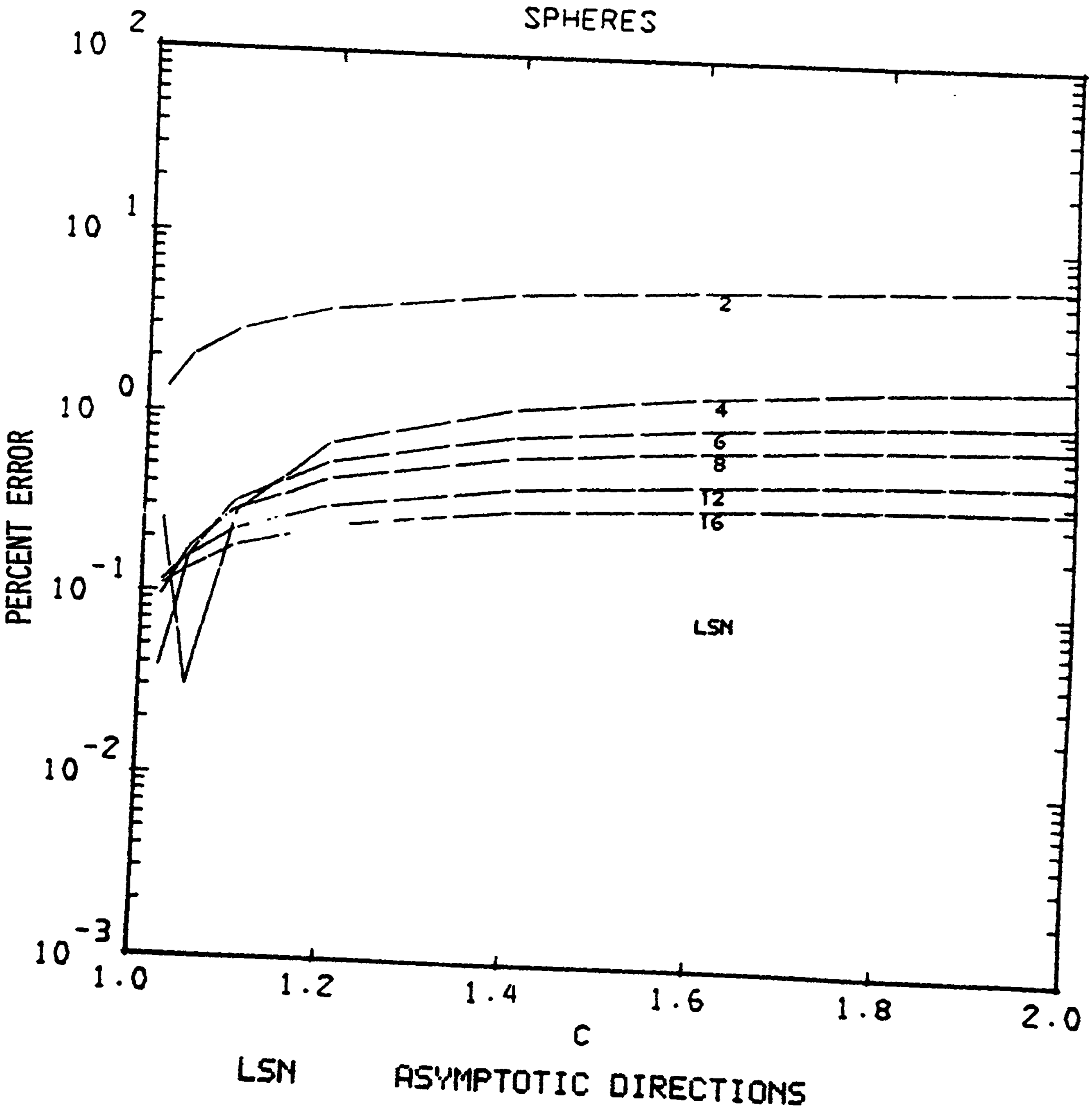

Figure 21. Percent error of critical radius for spheres versus c using LSN with asymptotic directions.

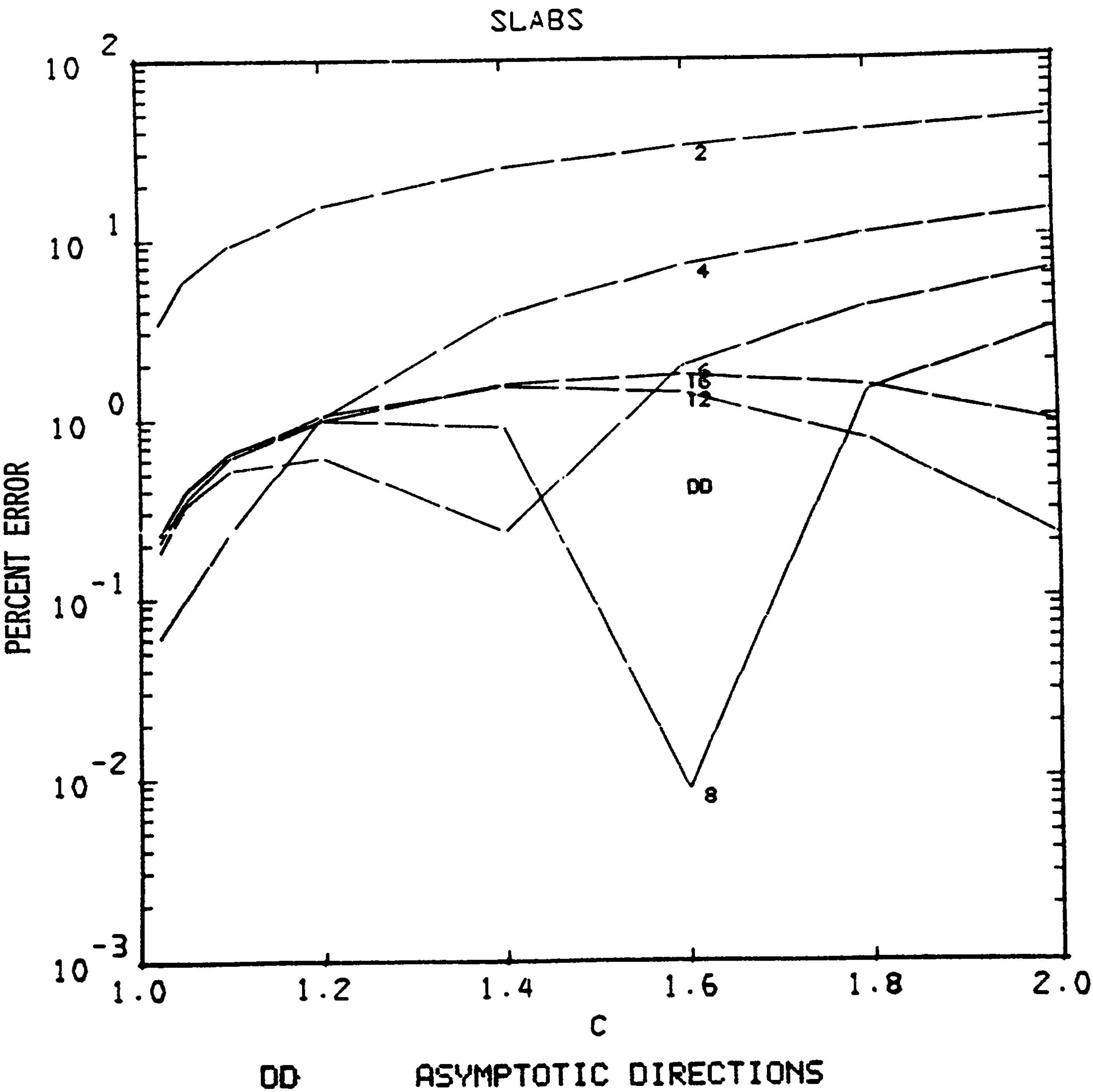

Figure 22. Percent error of critical half thickness for slabs versus c using DD with asymptotic directions.

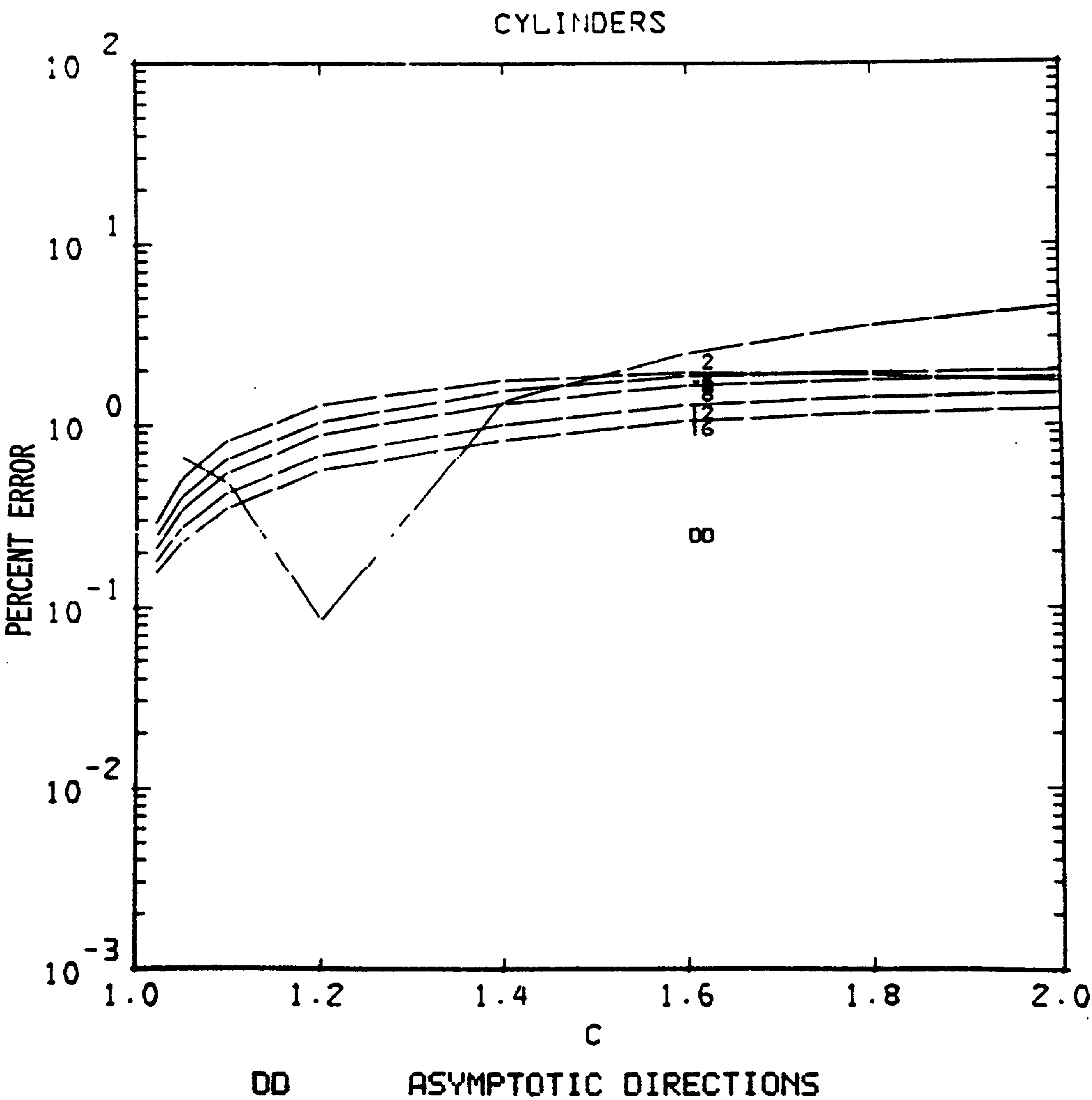

Figure 23. Percent error of critical radius for cylinders versus c using DD with asymptotic directions.

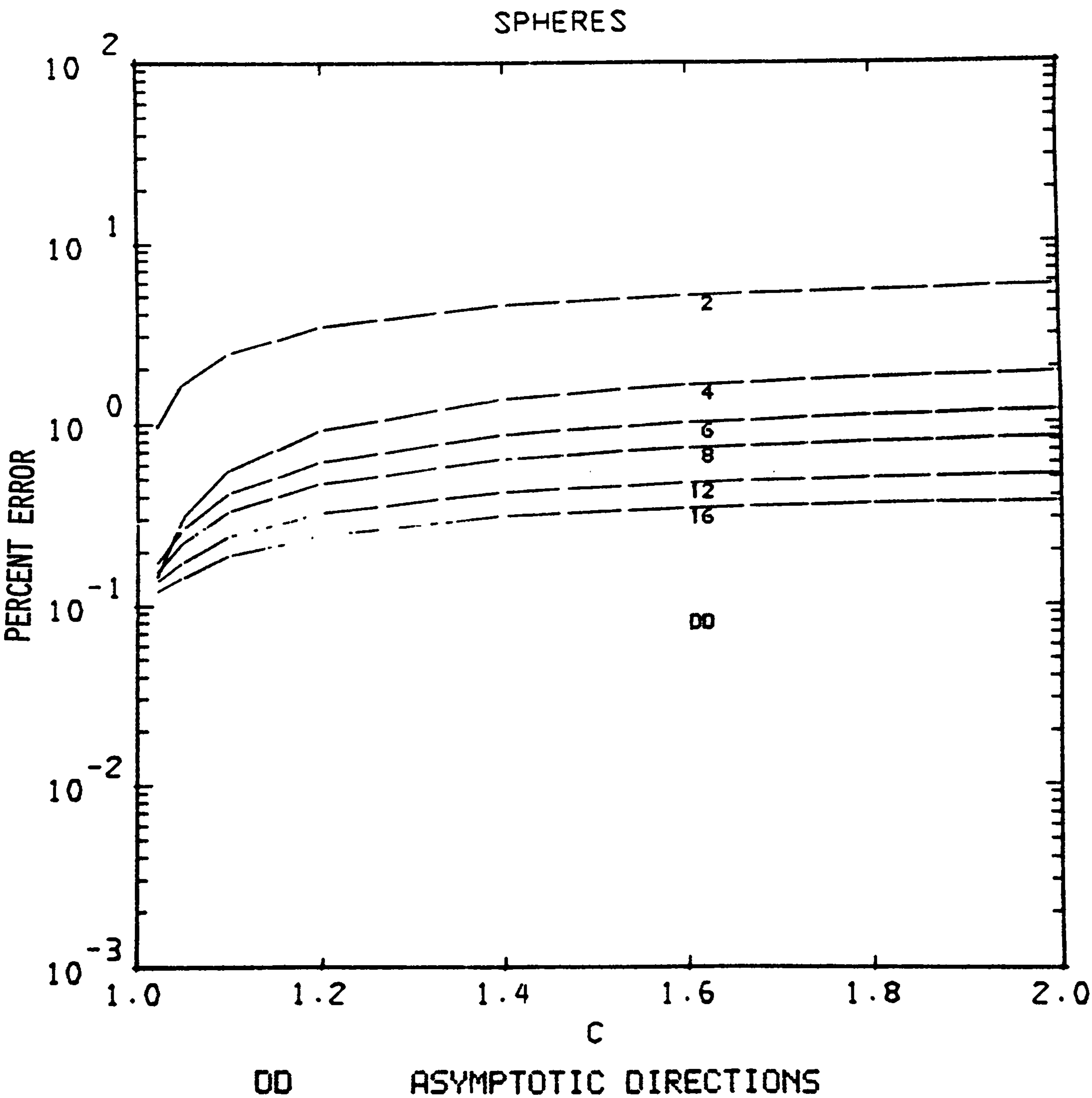

Figure 24. Percent error of critical radius for spheres versus c using DD with asymptotic directions.

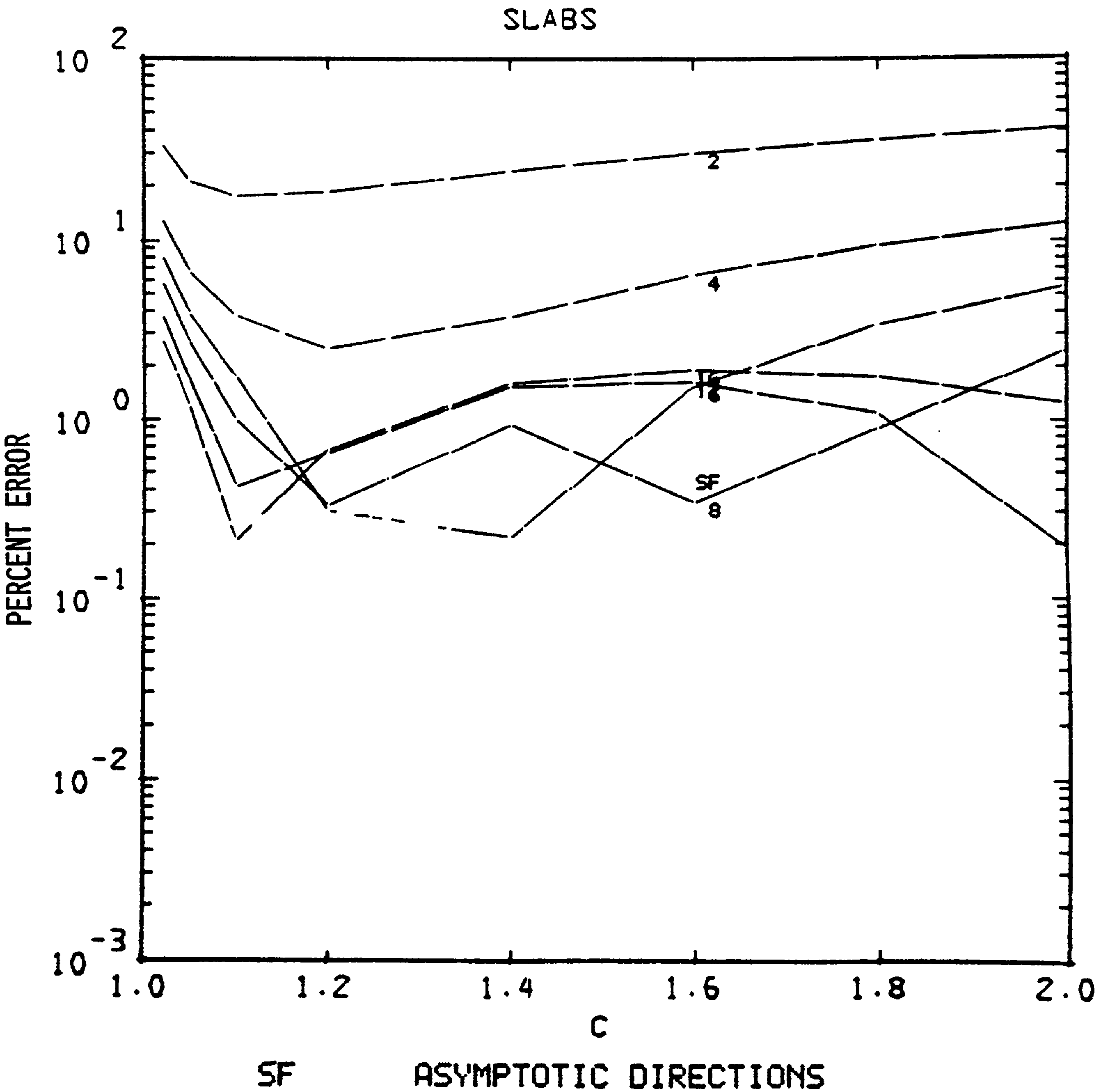

Figure 25. Percent error of critical radius for slabs versus c using SF with asymptotic directions.

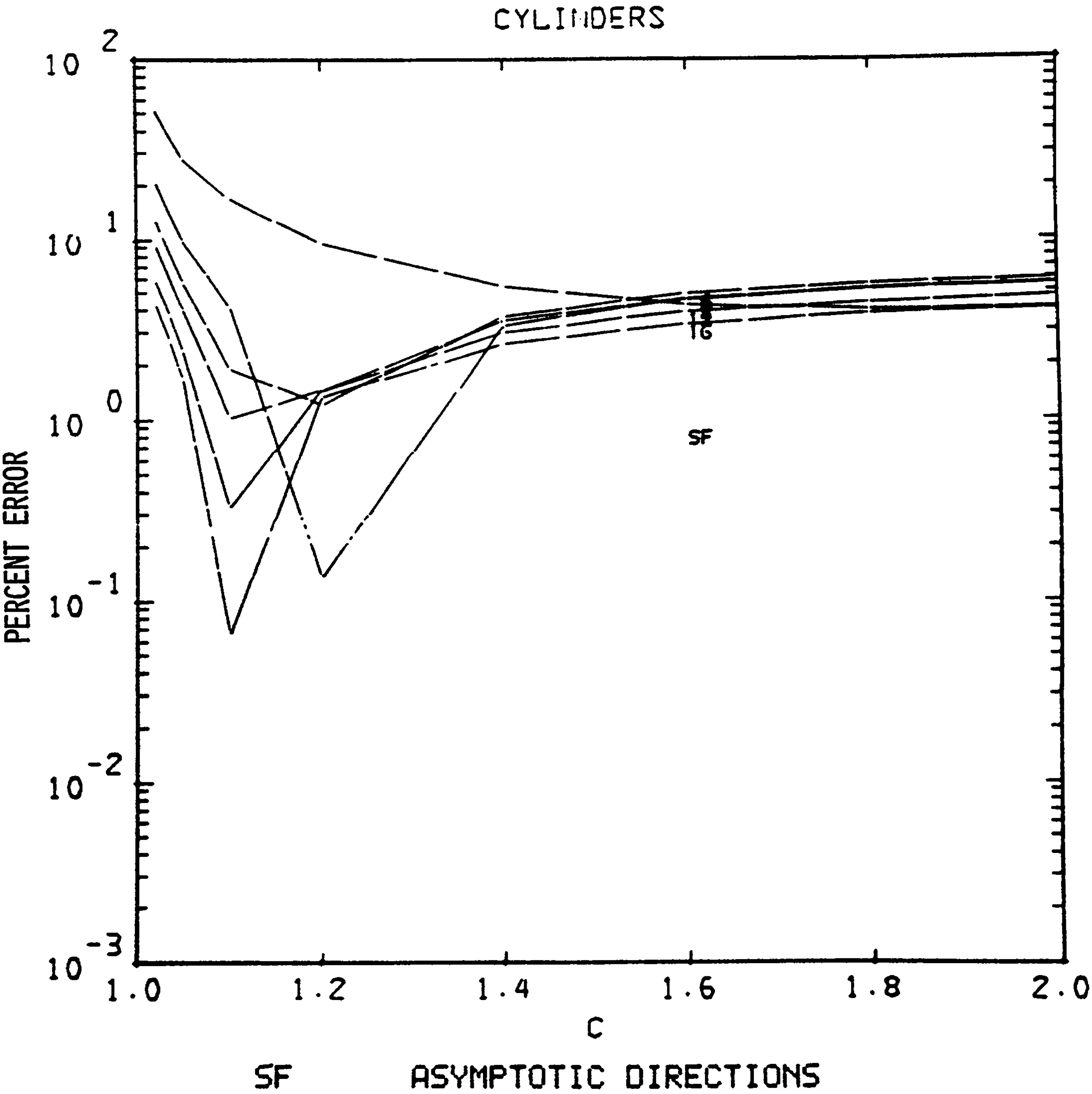

Figure 26. Percent error of critical radius for cylinders versus c using SF with asymptotic directions.

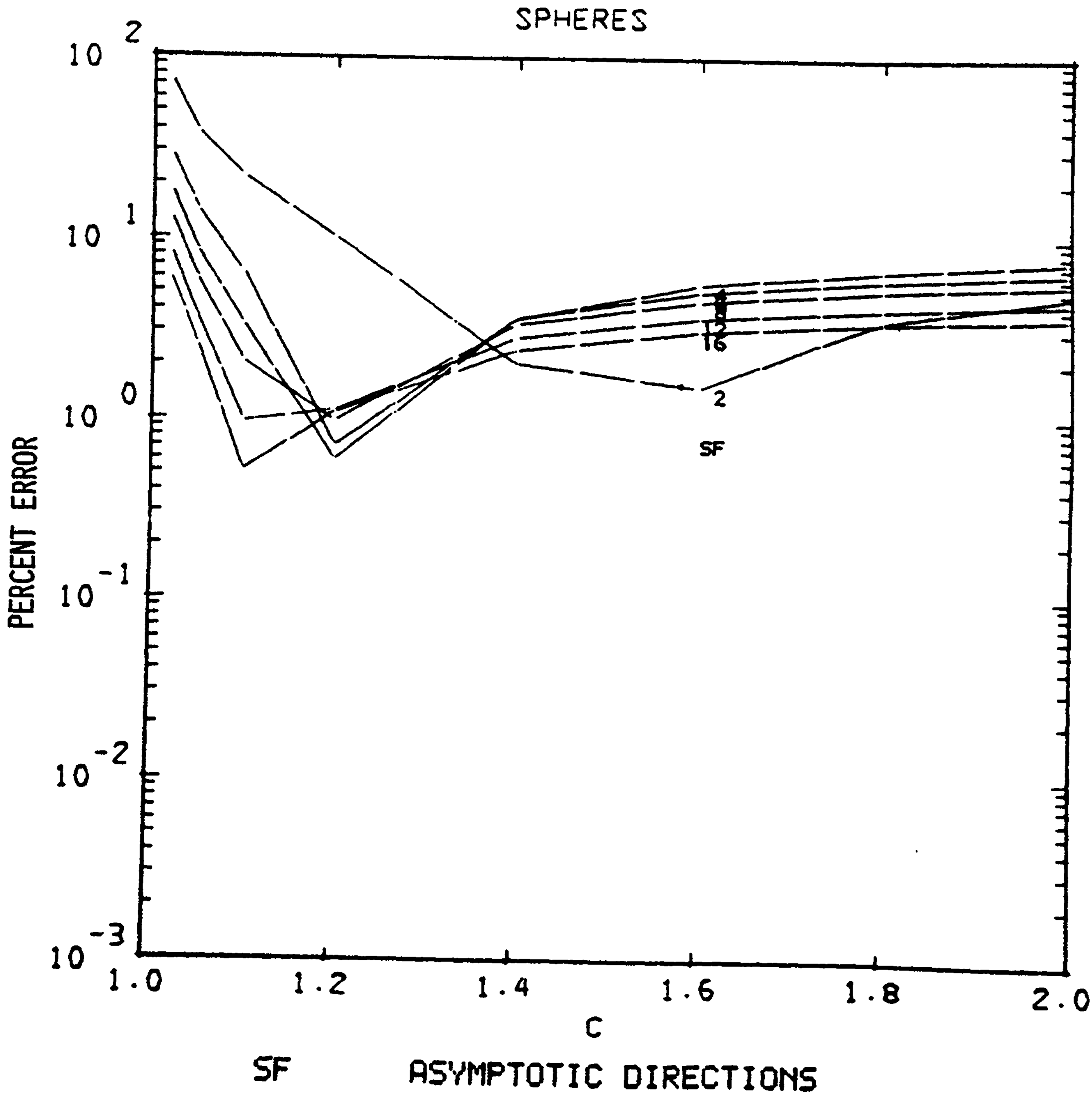

Figure 27. Percent error of critical radius for spheres versus c using SF with asymptotic directions.

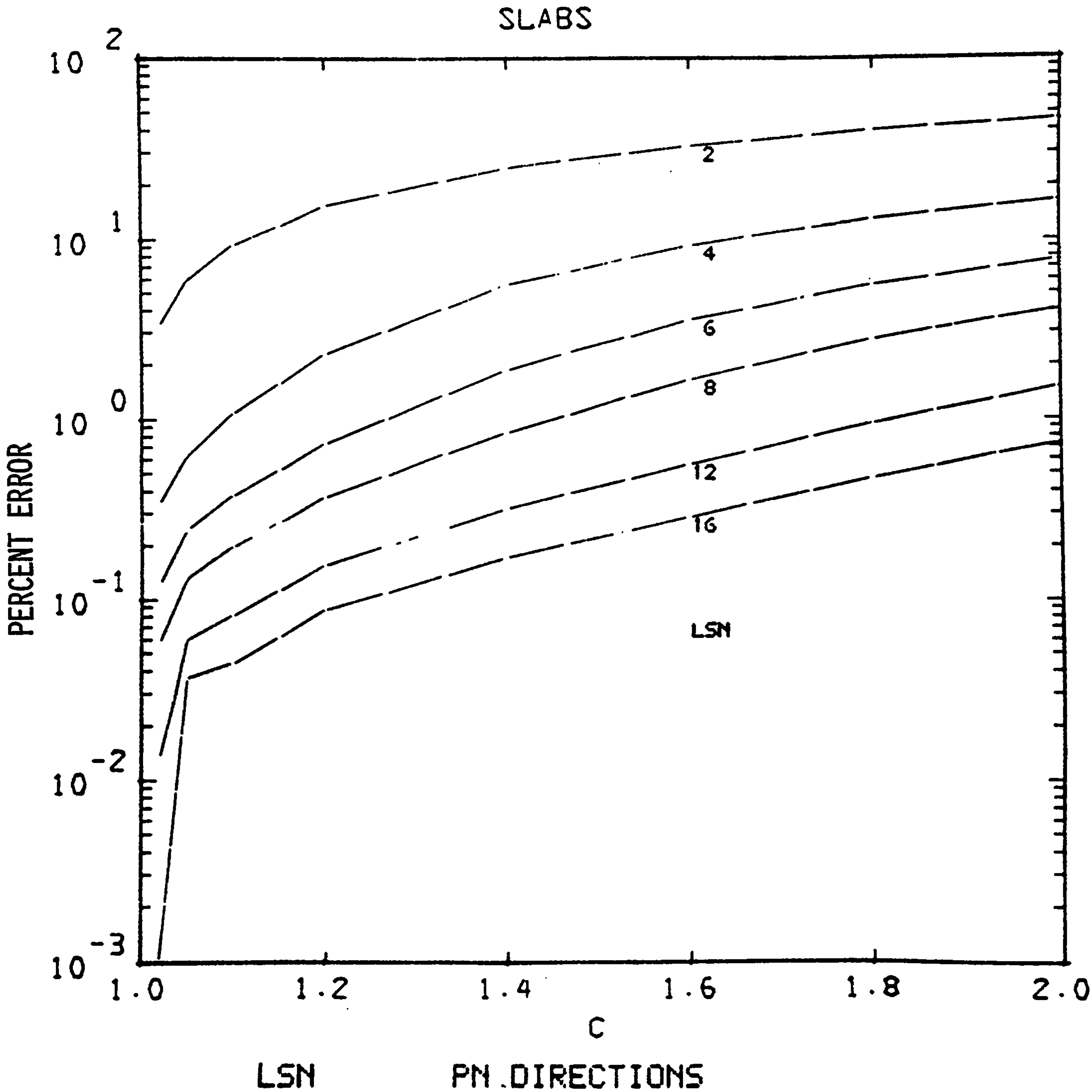

Figure 28. Percent error of critical half thickness for slabs versus c using LSN with P_n directions.

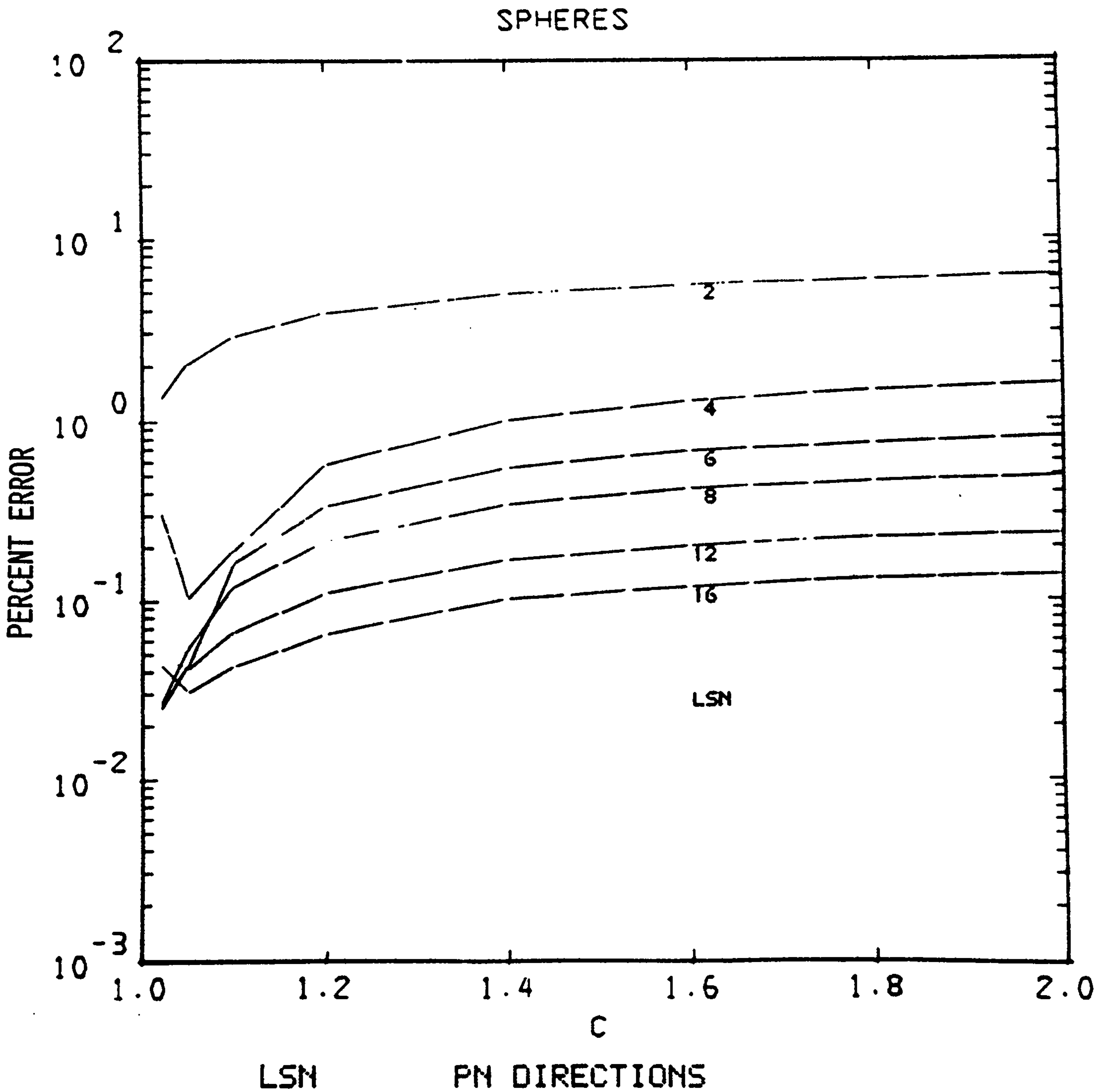

Figure 29. Percent error of critical radius for spheres versus c using LSN with P_n directions.

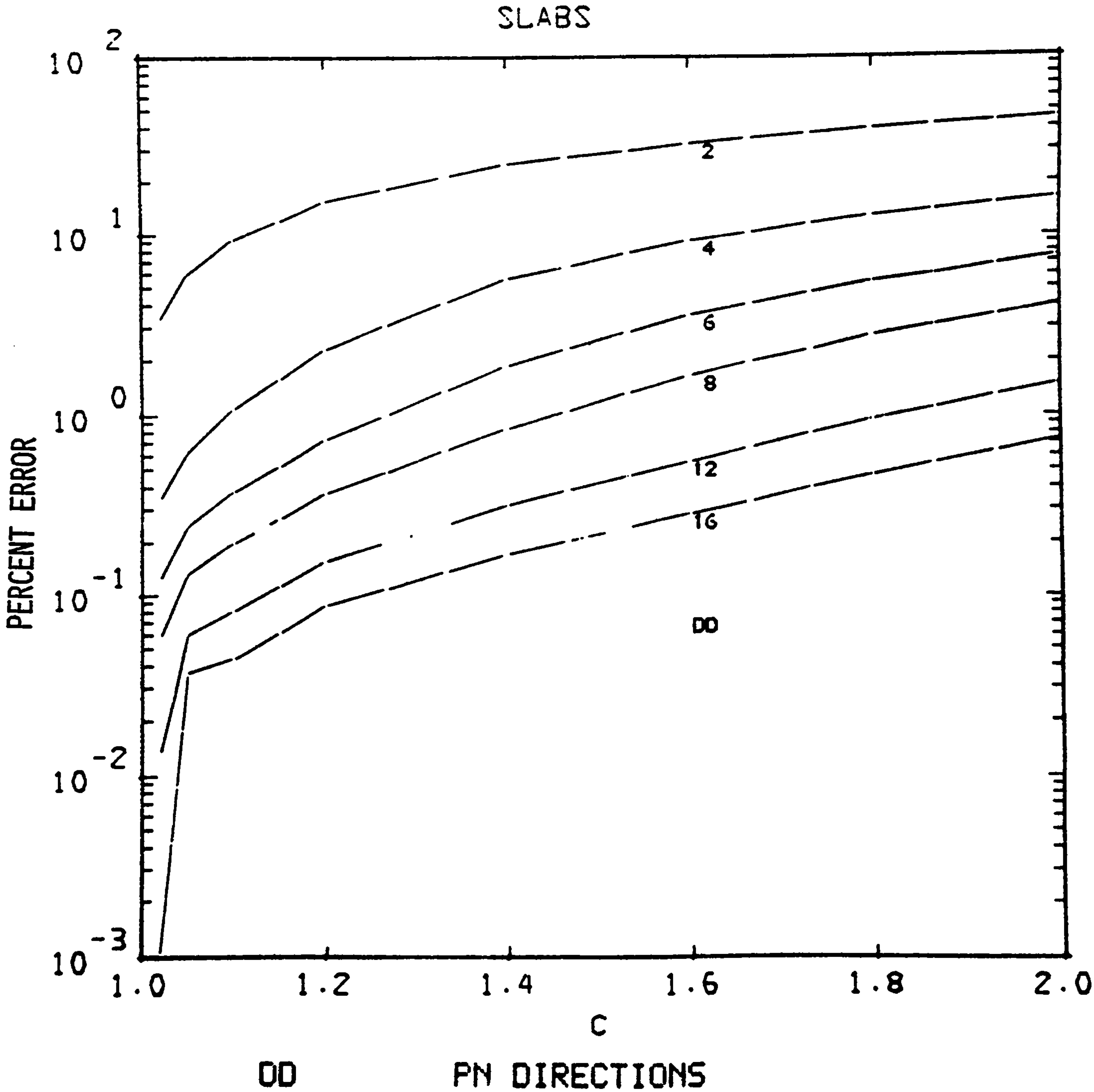

Figure 30. Percent error of critical half thickness for slabs versus c using DD with P_n directions.

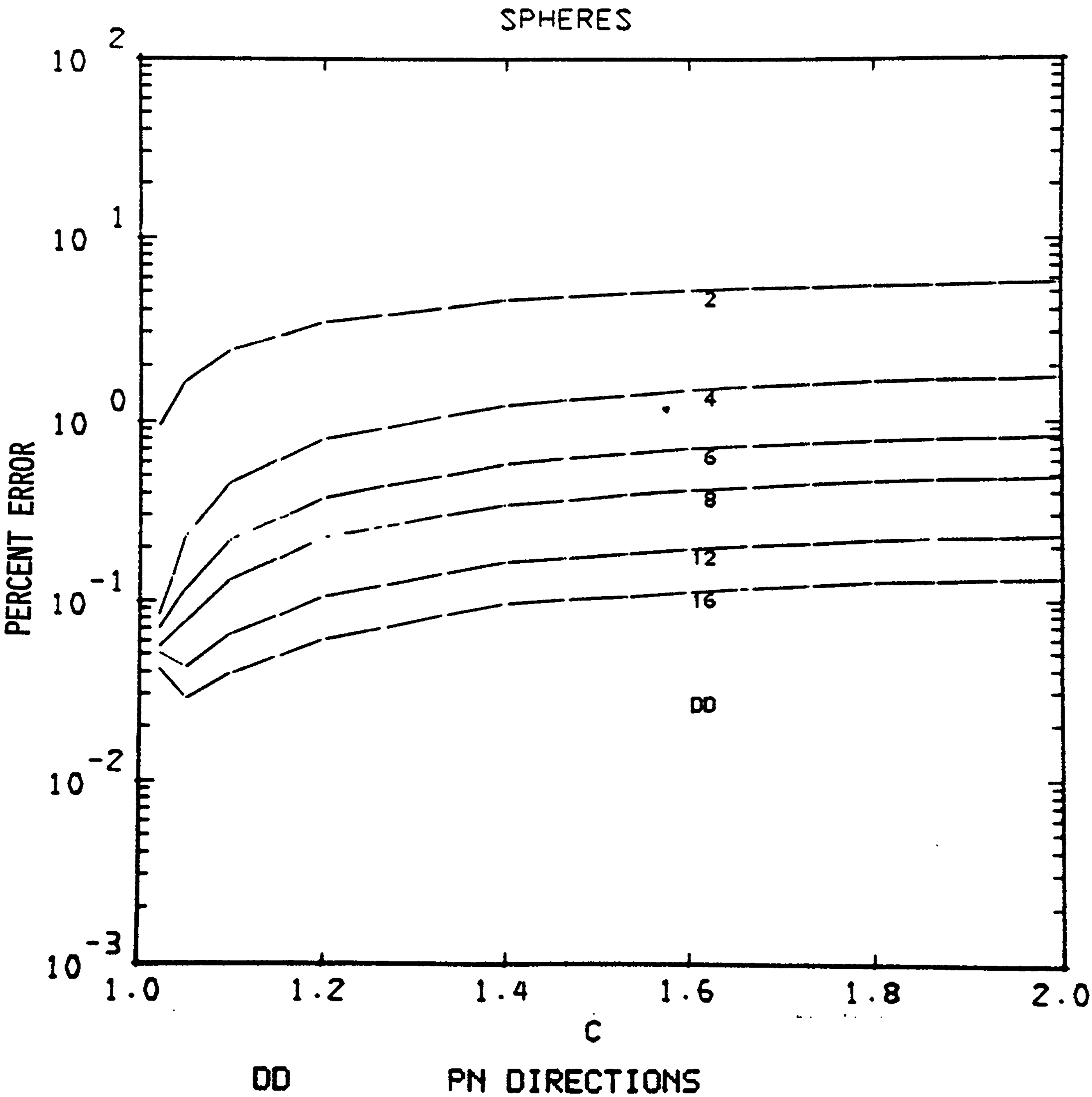

Figure 31. Percent error of critical radius for spheres versus c using DD with P_n directions.

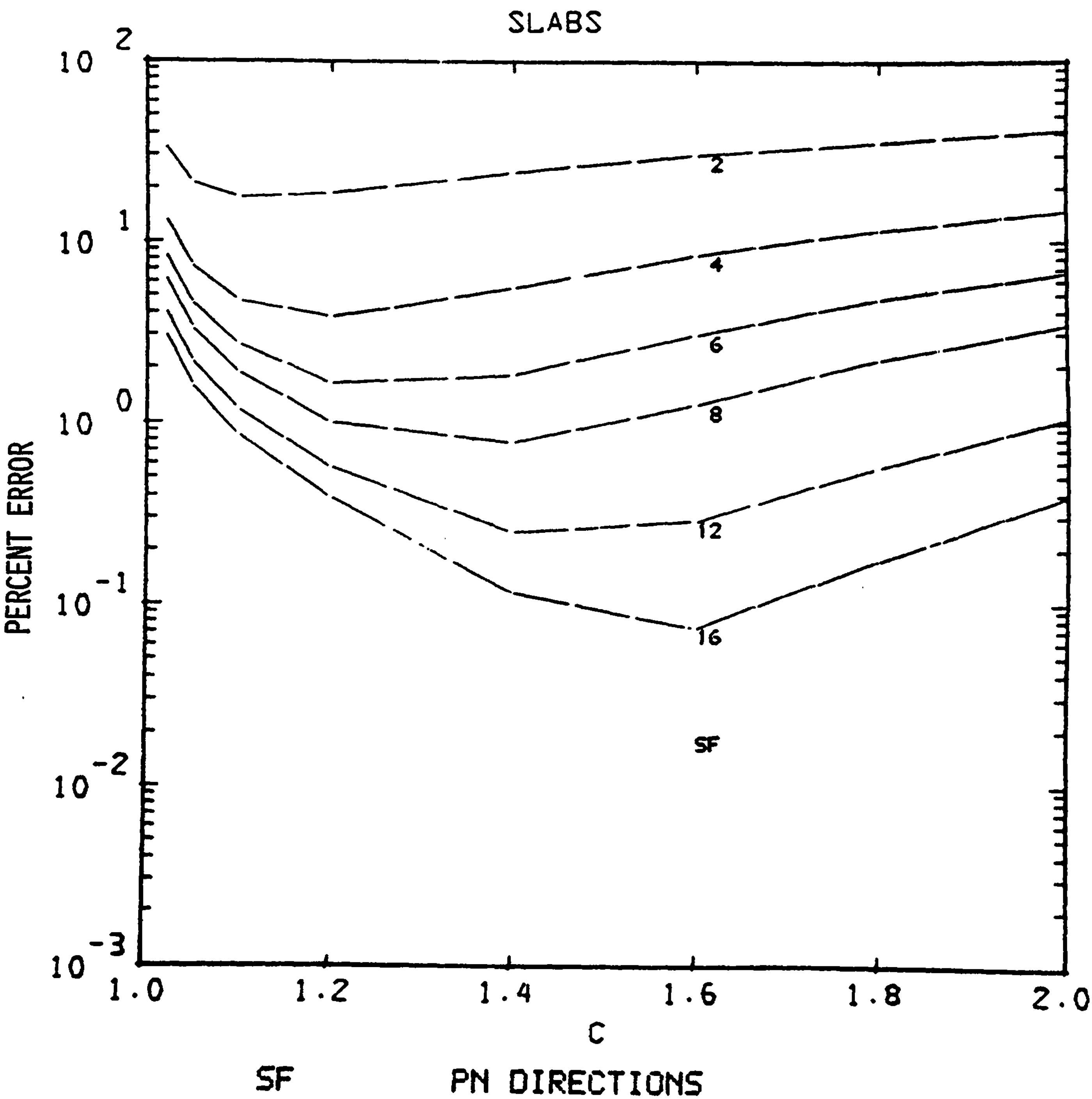

Figure 32. Percent error of critical half thickness for slabs versus c using SF with P_n directions.

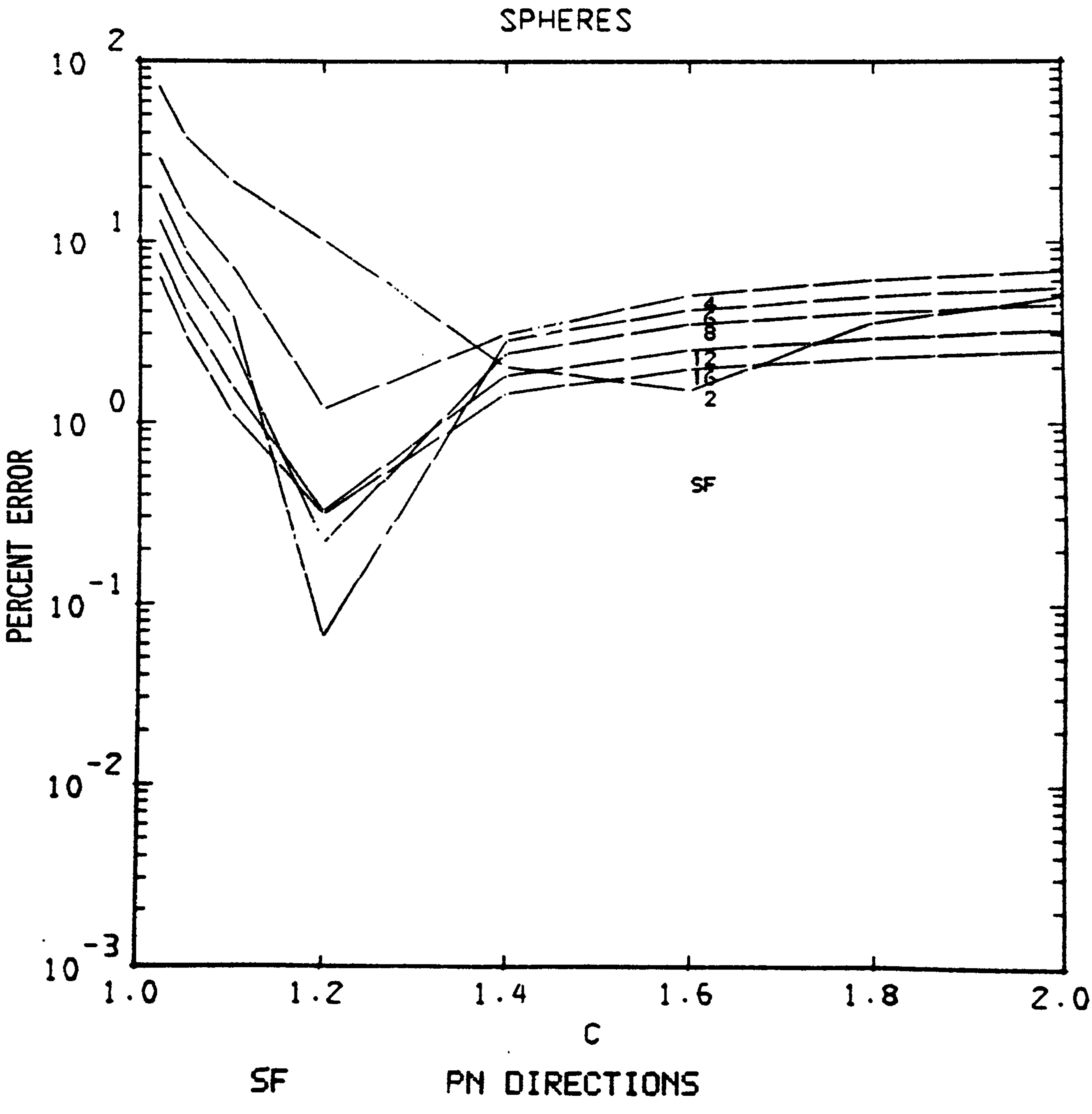

Figure 33. Percent error of critical radius for spheres versus c using SF with P_n directions.

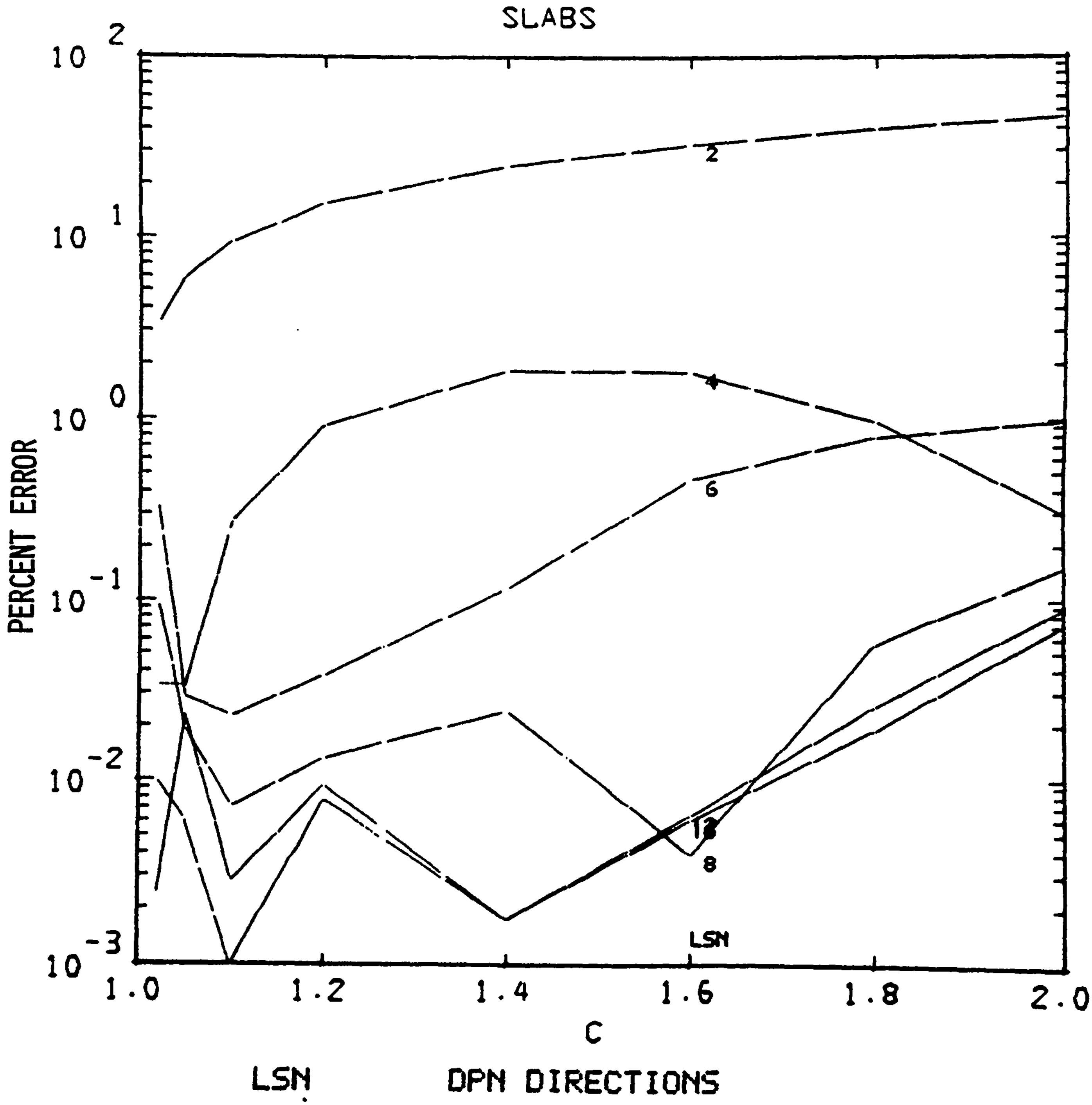

Figure 34. Percent error of critical half thickness for slabs versus c using LSN with DP_n directions.

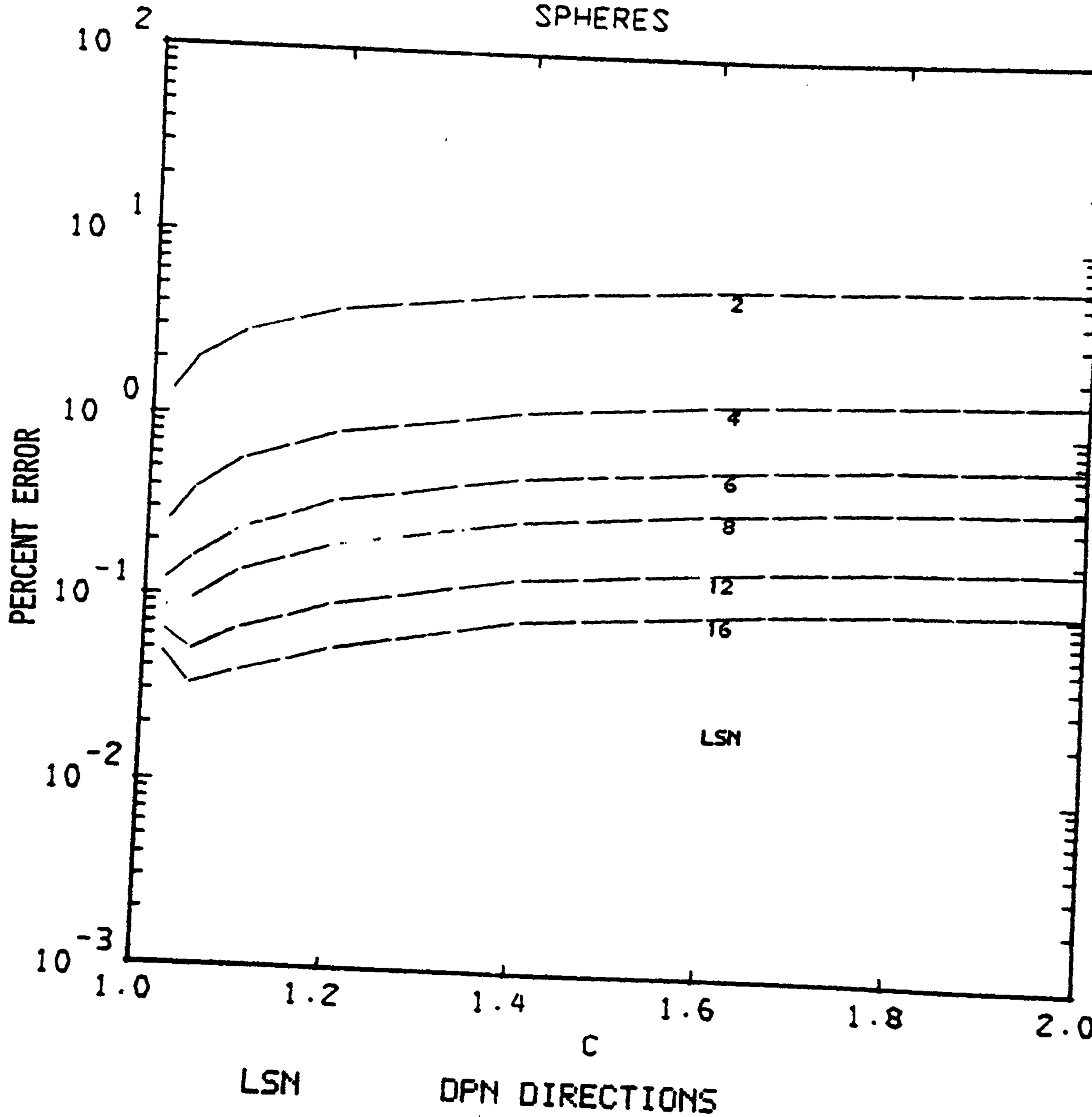

Figure 35. Percent error of critical radius for spheres versus c using LSN with DP_n directions.

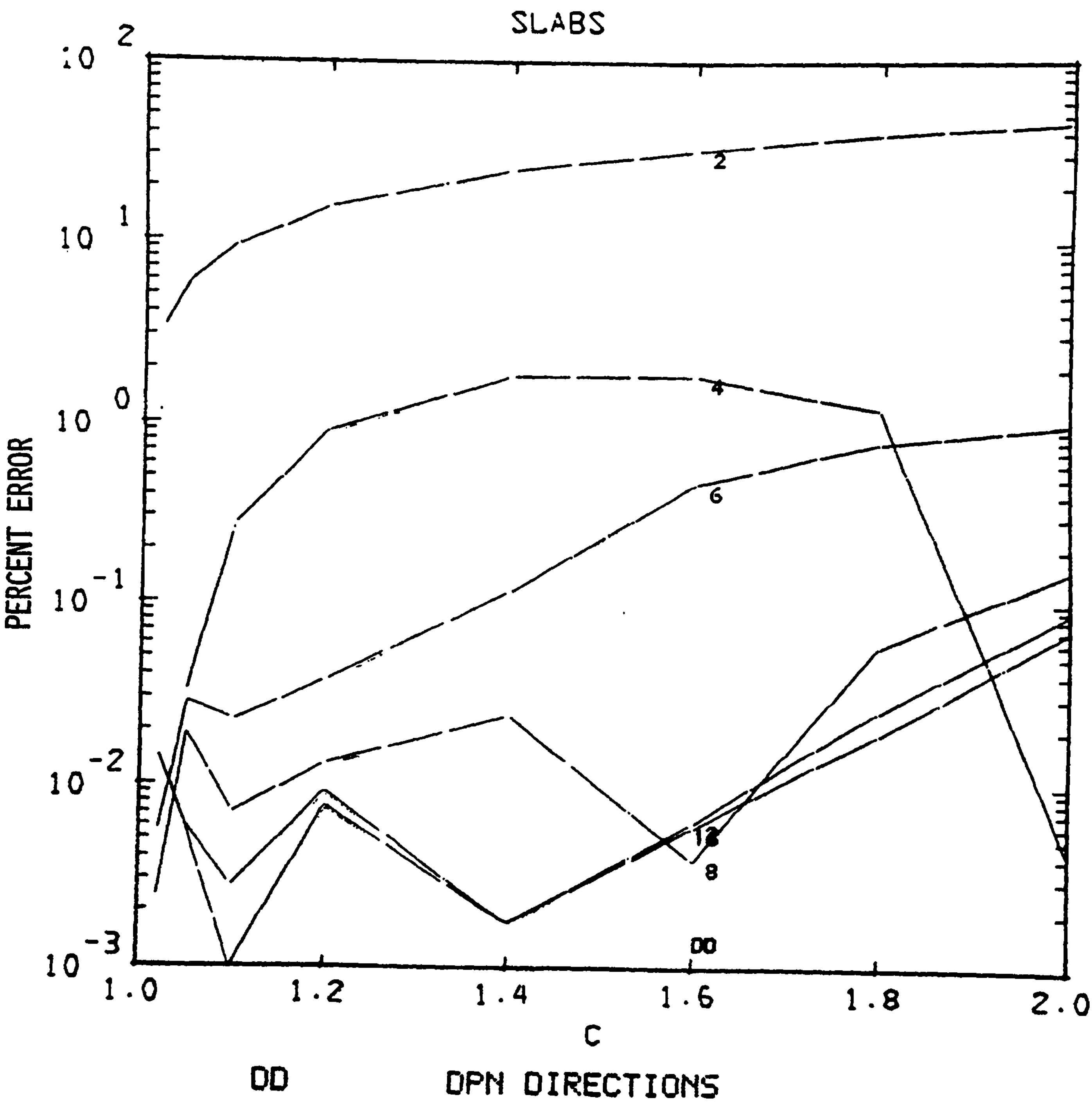

Figure 36. Percent error of critical half thickness for slabs versus c using DD with DP_n directions.

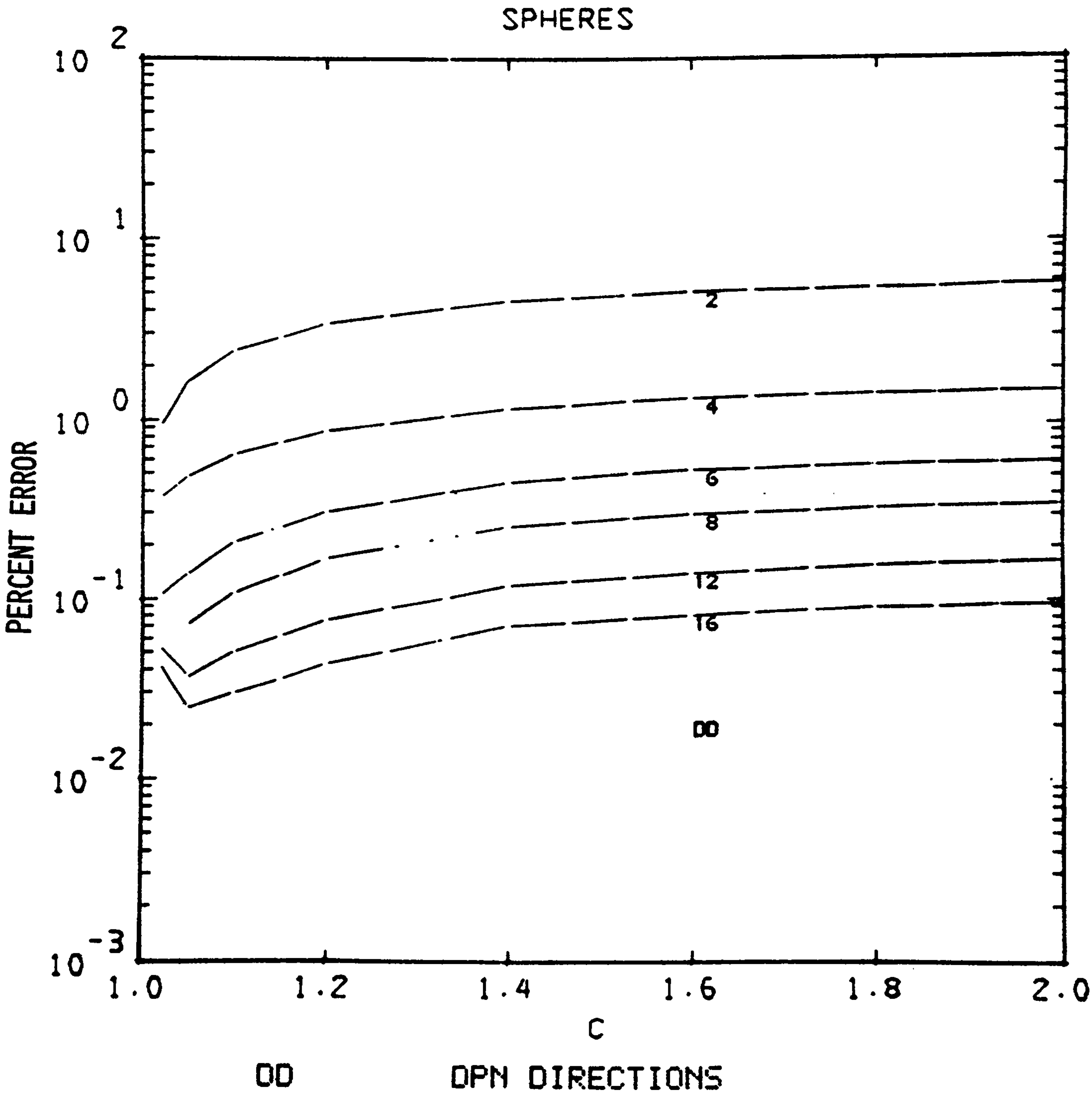

Figure 37. Percent error of critical radius for spheres versus c using DD with DP_n directions.

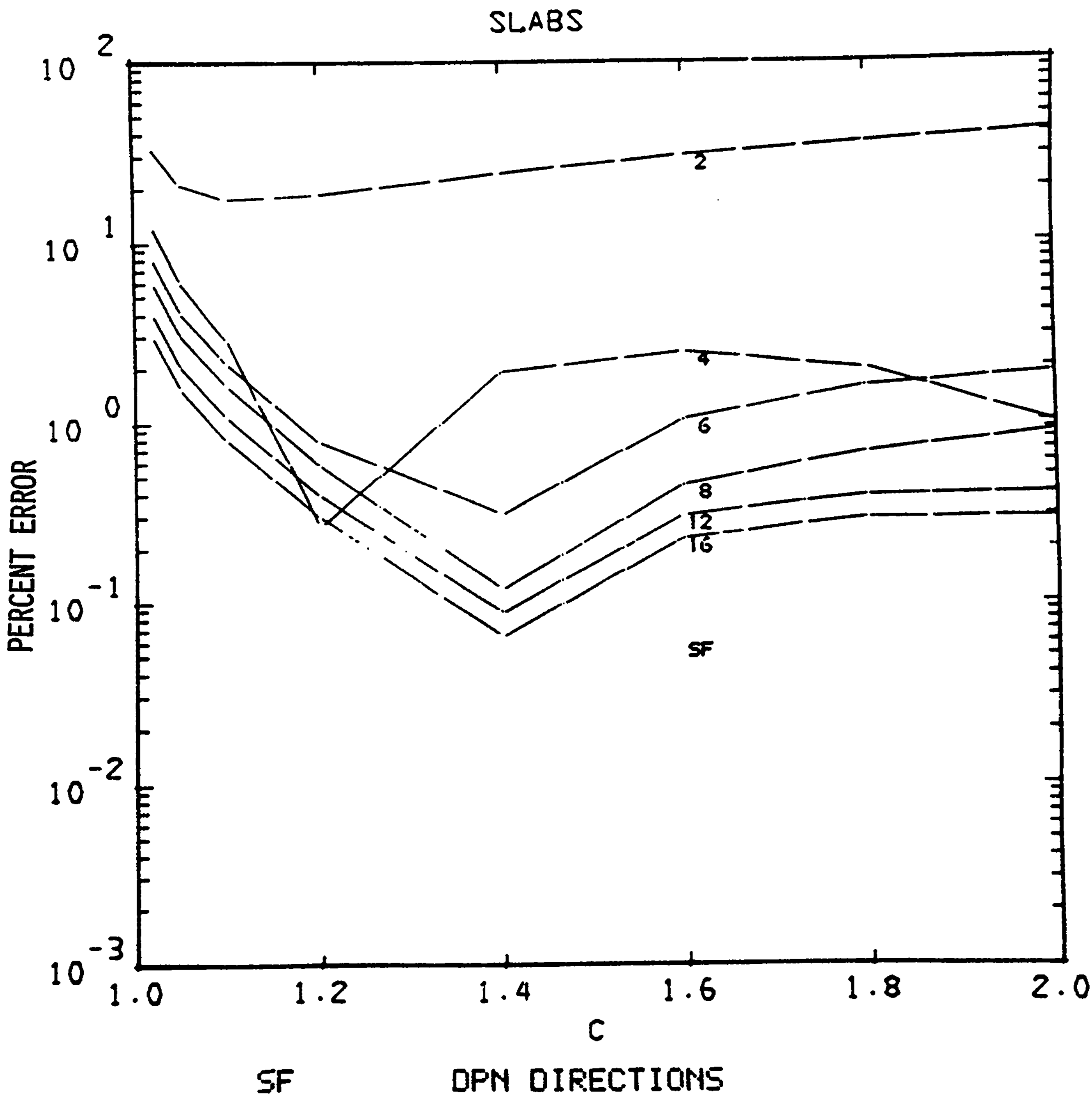

Figure 38. Percent error of critical half thickness for slabs versus c using SF with DP_n directions.

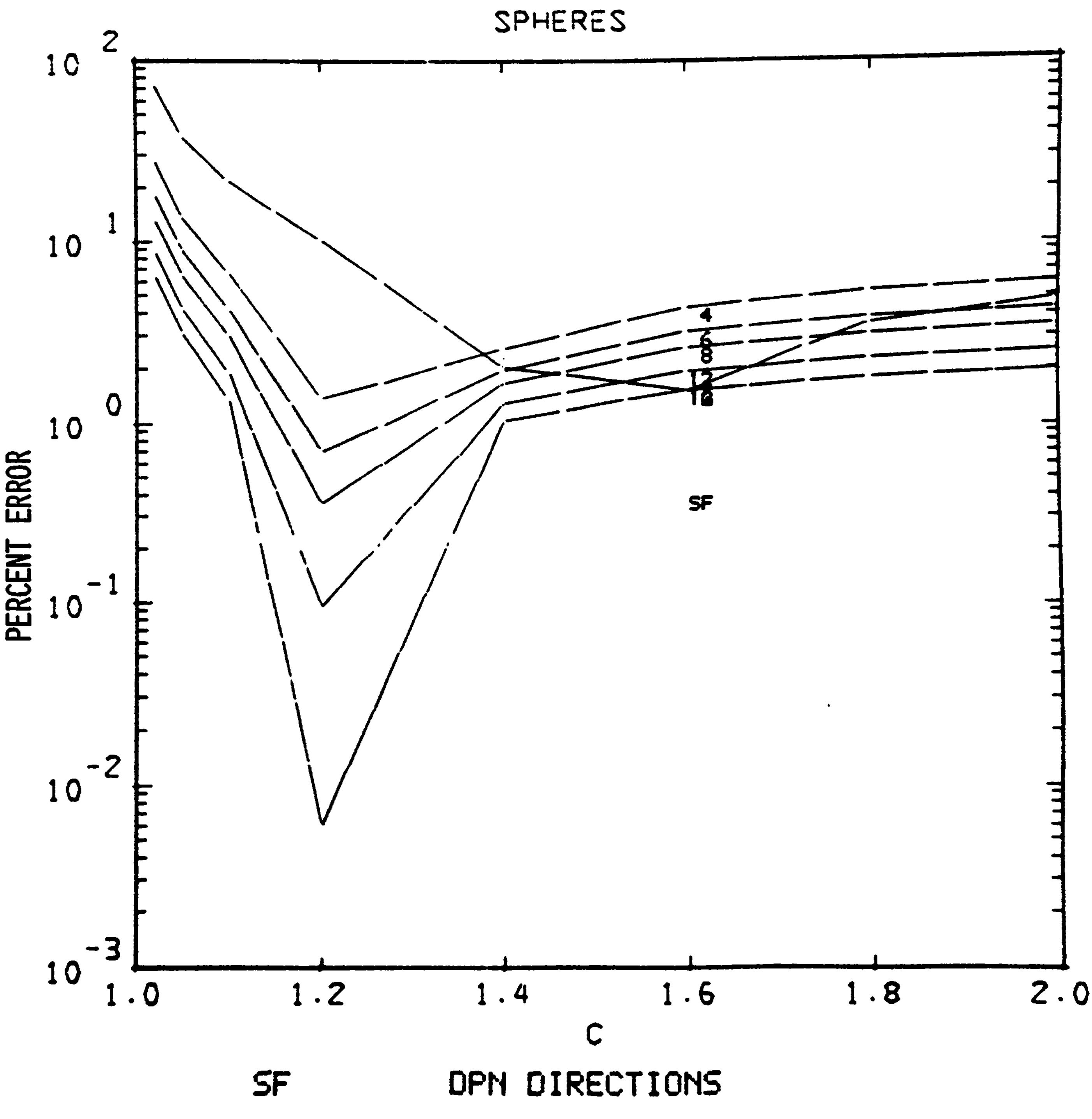

Figure 39. Percent error of critical half thickness for spheres versus c using SF with DP_n directions.

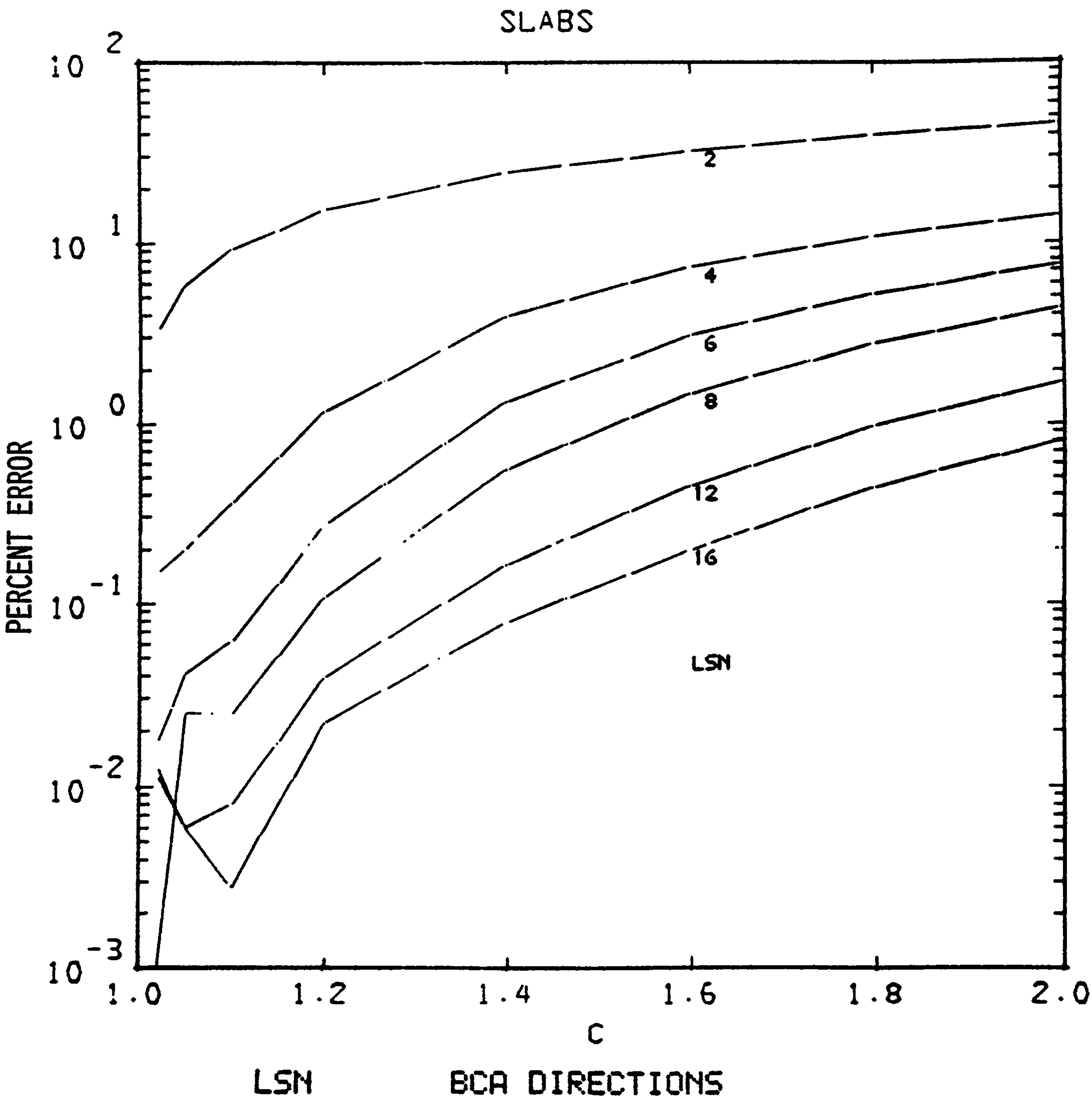

Figure 40. Percent error of critical half thickness for slabs versus c using LSN with CA directions.

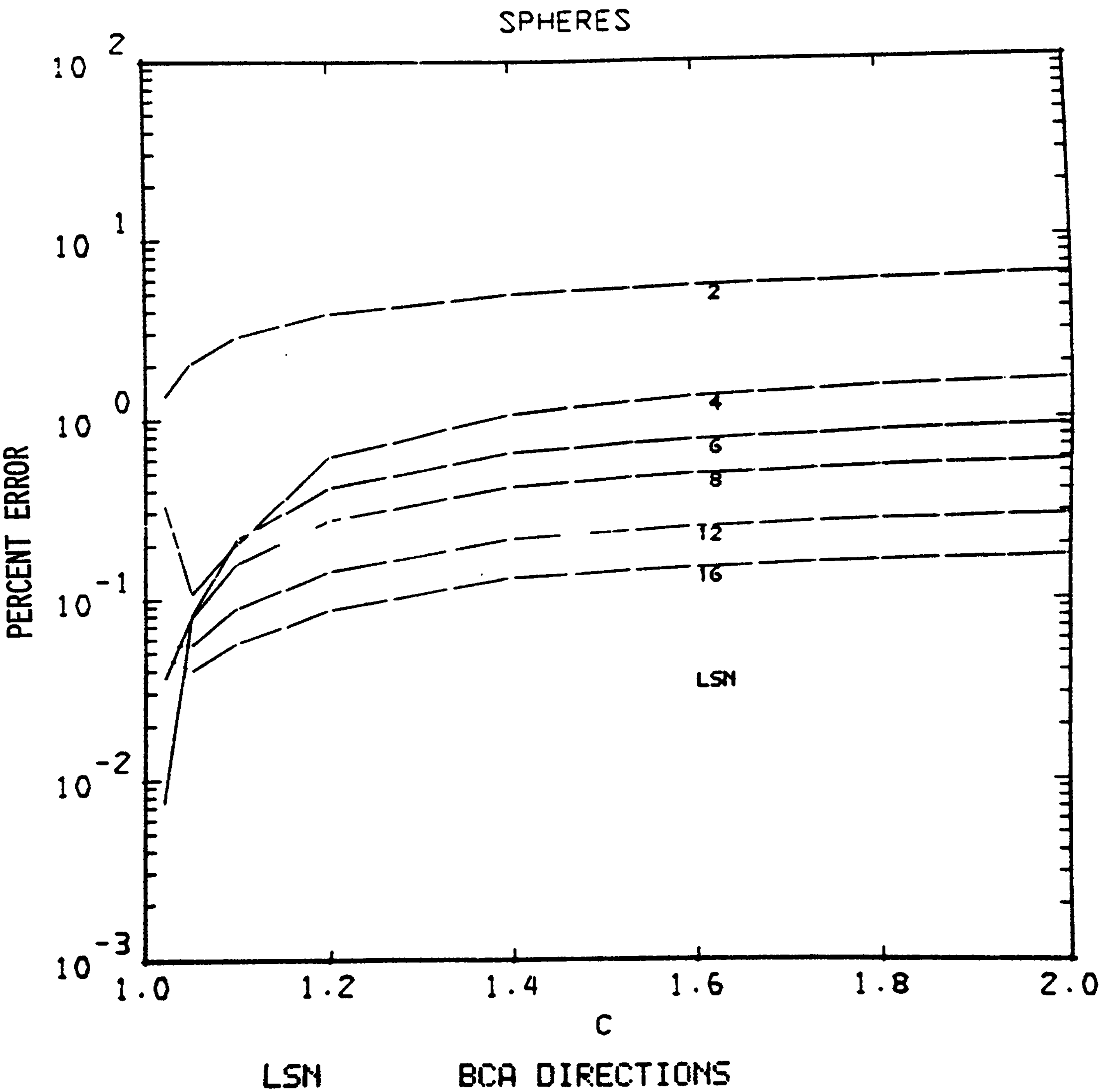

Figure 41. Percent error of critical radius for spheres versus c using LSN with CA directions.

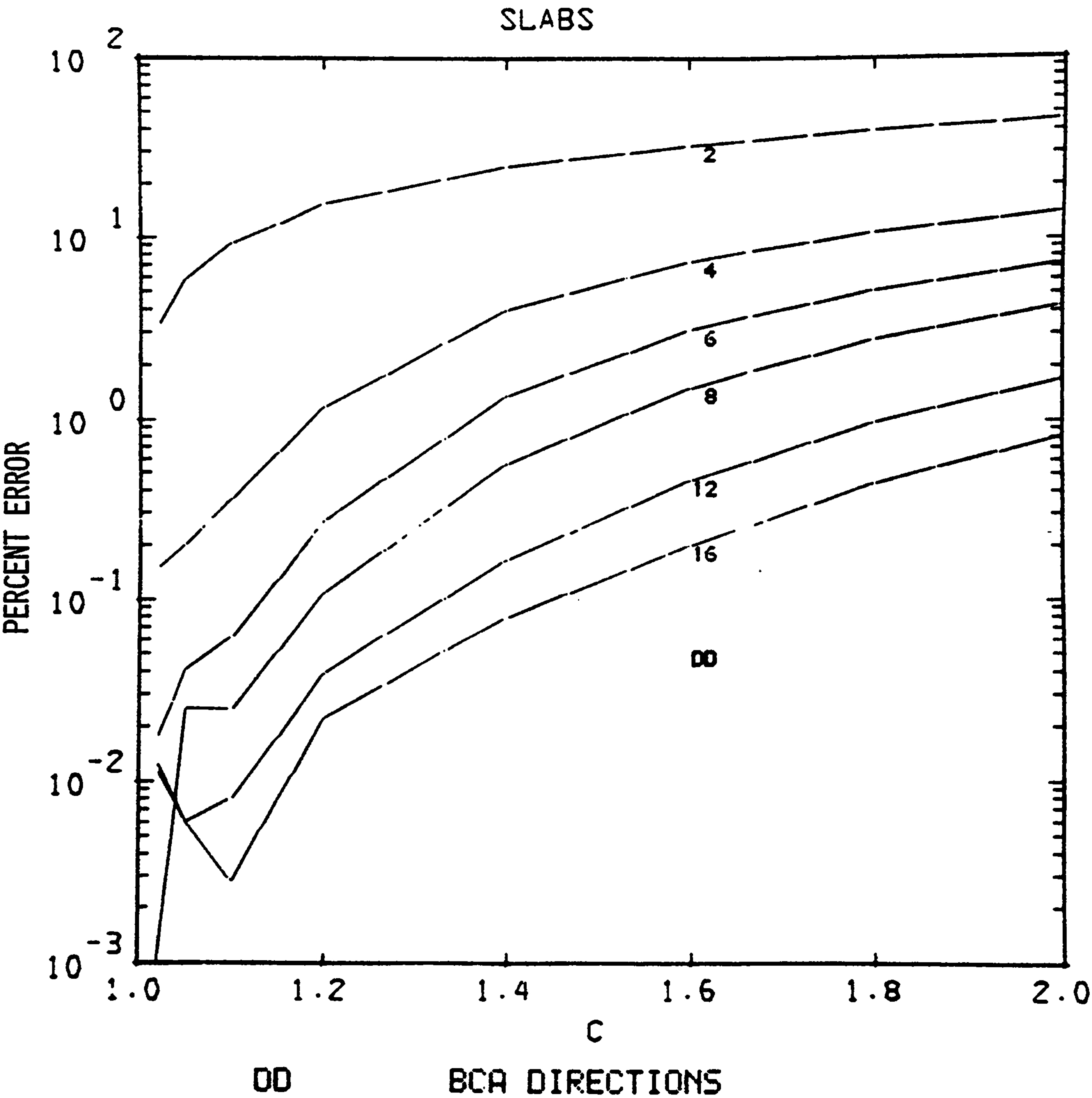

Figure 42. Percent error of critical half thickness for slabs versus c using DD with CA directions.

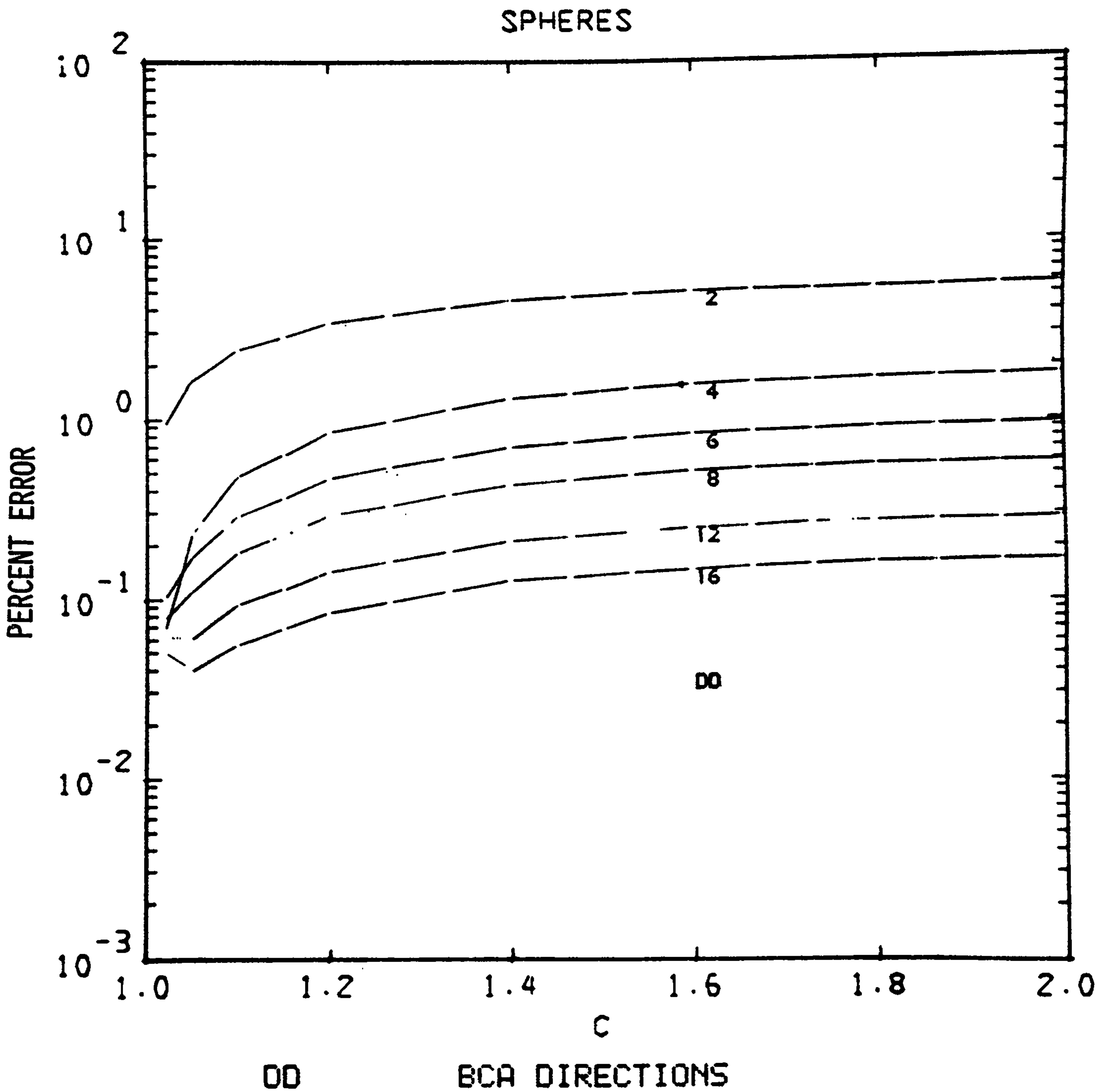

Figure 43. Percent error of critical radius for spheres versus c using DD with CA directions.

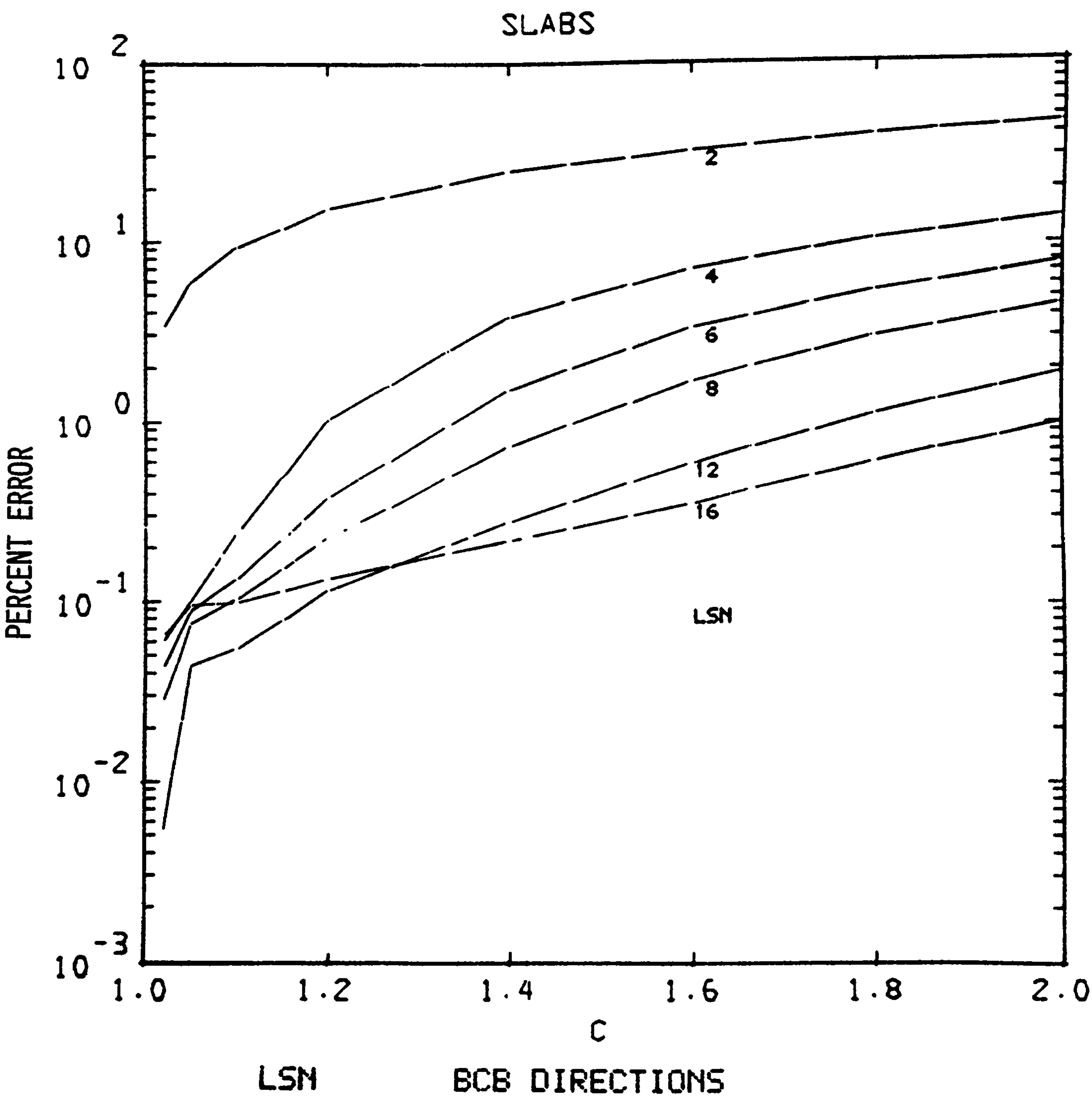

Figure 44. Percent error of critical half thickness for slabs versus c using LSN with CB directions.

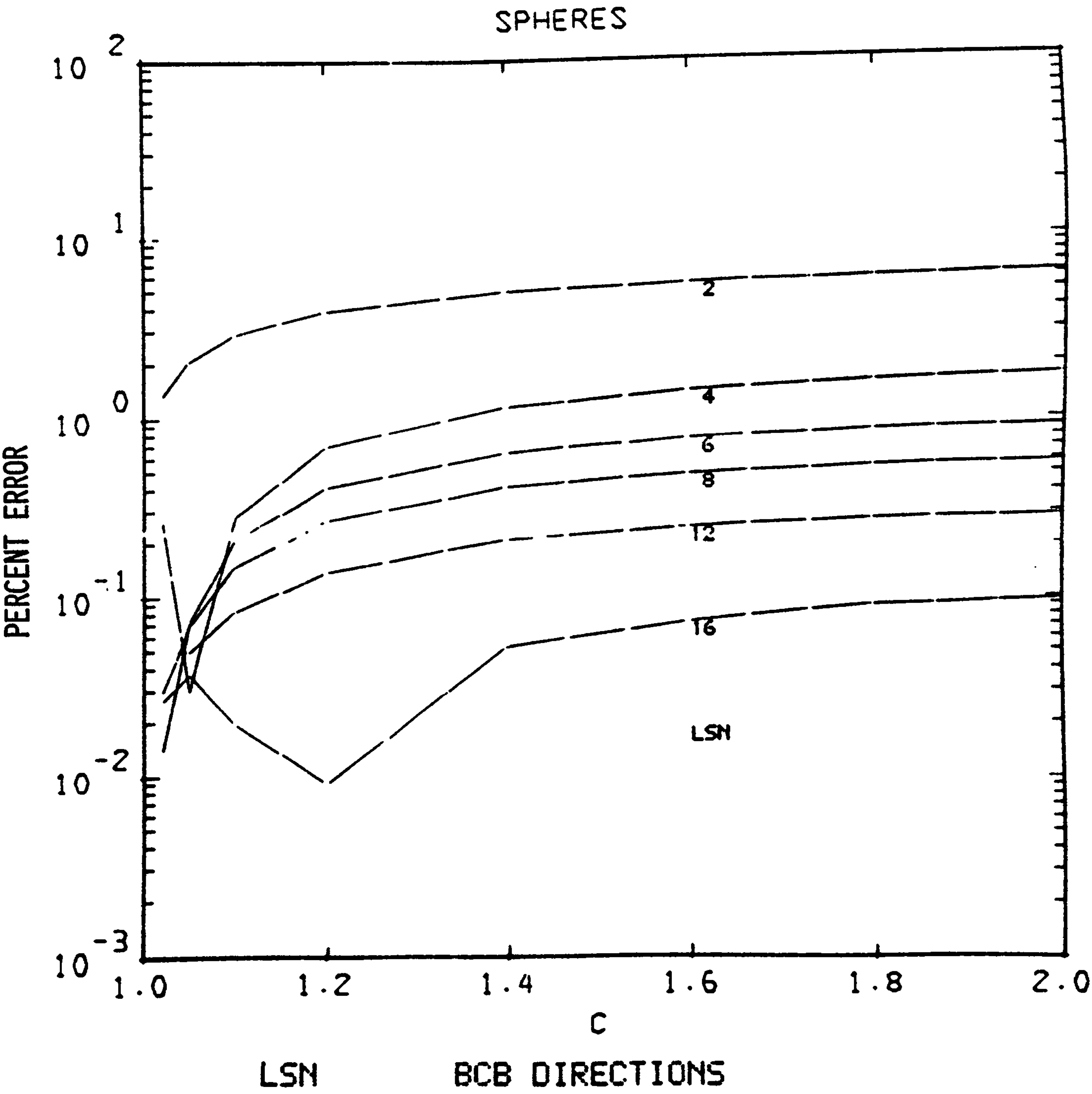

Figure 45. Percent error of critical radius for spheres versus c using LSN with CB directions.

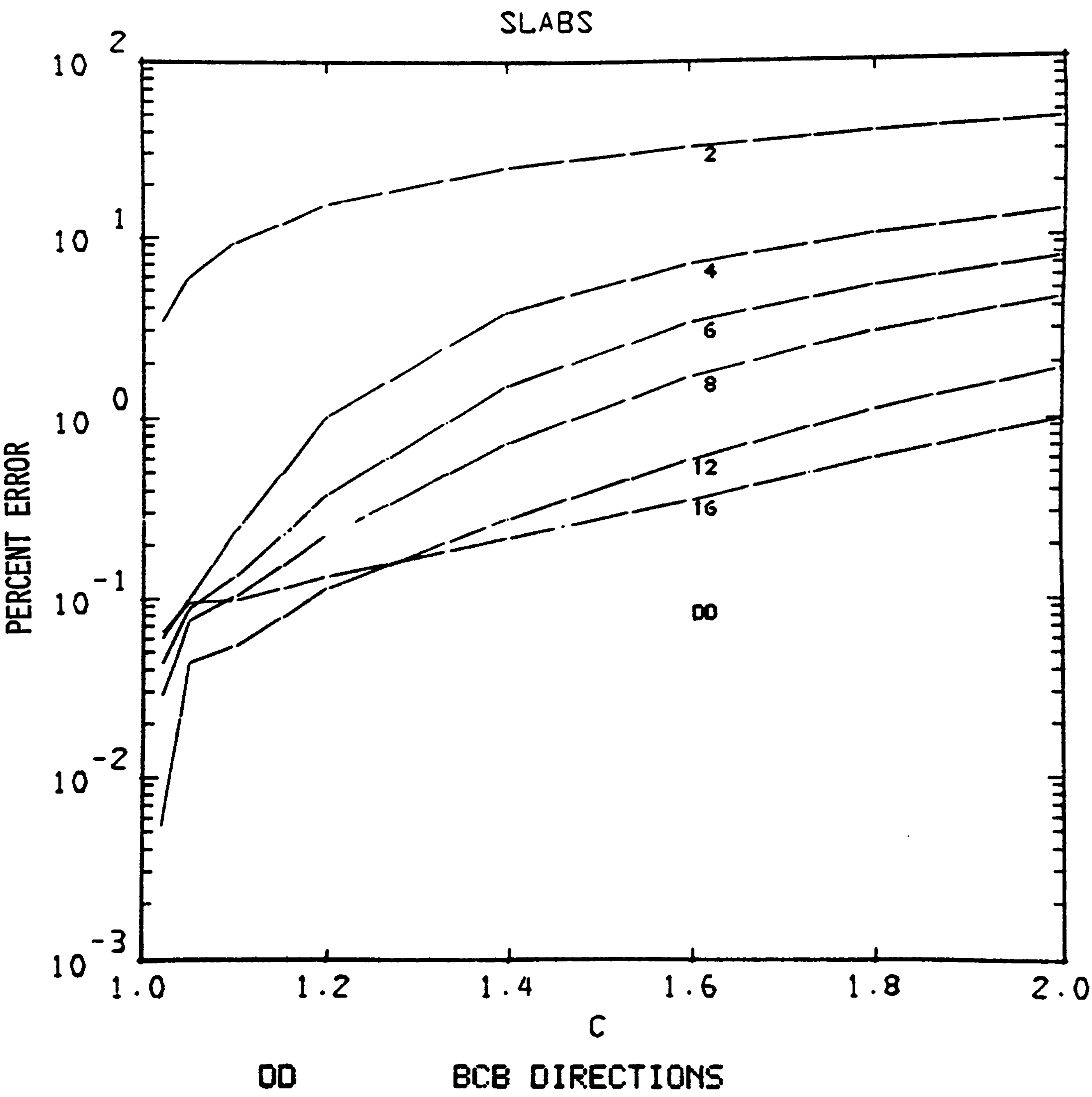

Figure 46. Percent error of critical half thickness for slabs versus c using DD with CB directions.

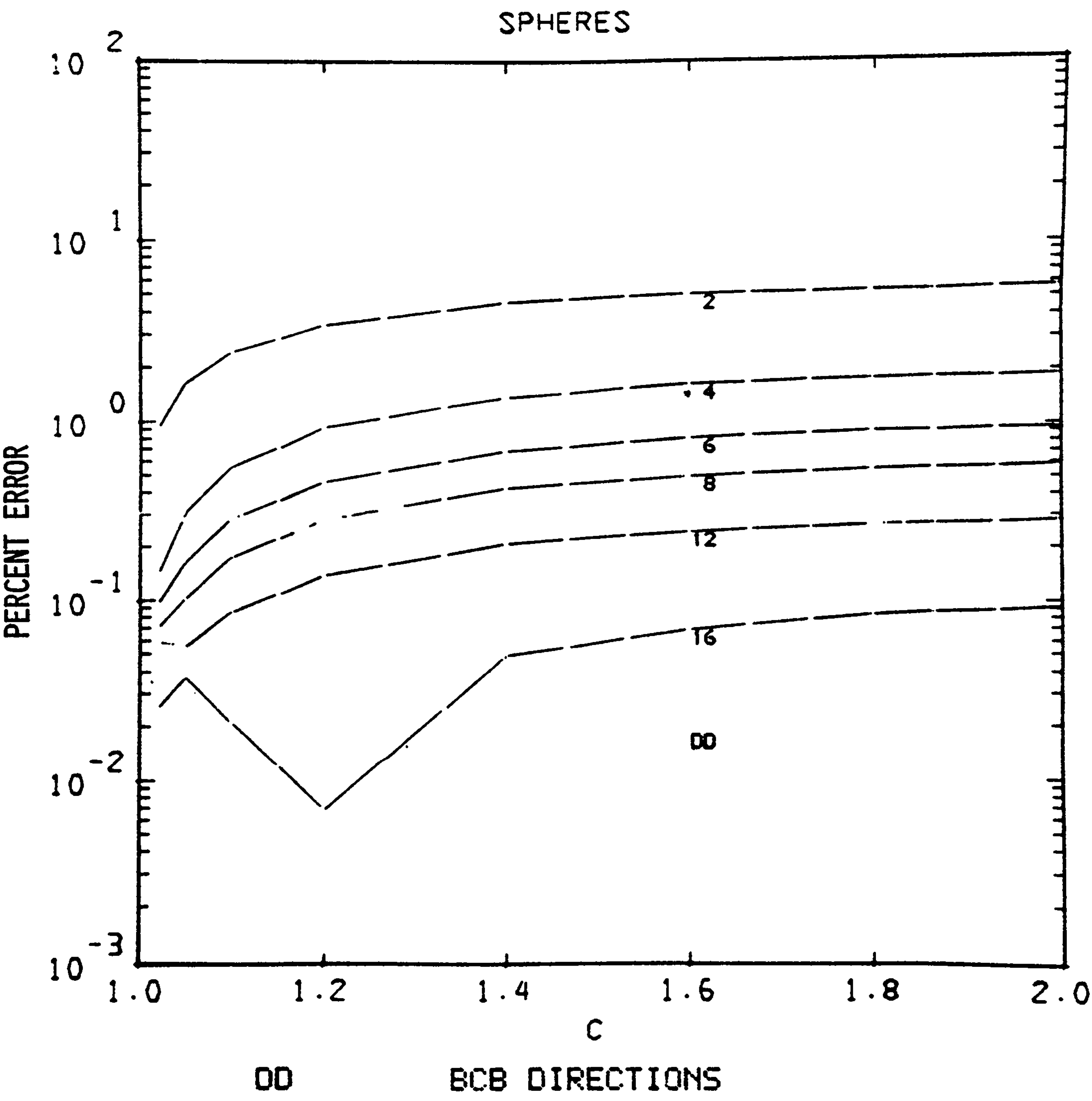

Figure 47. Percent error of critical radius for spheres versus c using DD with CB directions.

0 SLABS

AM: The LSN (10) and DD (13) are essentially identical for all n and all c. The SF (16) is less accurate than LSN (10) or DD (13) for c less than 1.4, after which for higher c values, SF is slightly more accurate than either.

AD: The LSN (19) and DD (22) are essentially identical for all n and all c. For n = 2, 4, and 6, SF (25) is slightly more accurate than LSN (10) or DD (22) for c greater than 1.4. For n = 8, 12, and 16, SF (25) is more accurate than LSN (19) or DD (22) for c between 1.2 and 1.6; otherwise, it is less accurate than either.

PN: The LSN (28) and DD (30) are essentially identical for all n and all c. The SF (32) is less accurate than LSN (28) or DD (30) for c less 1.4; and n = 2, 4, 6, and 8, SF is more accurate than LSN (28) or DD (30) for c greater than 1.4 for all n.

DPN: The LSN (34) and DD (36) are essentially identical for all n and all c, except for n = 4 at c = 1.8 and 2.0, where DD (36) is more accurate than LSN (34). The SF (38) is less accurate than LSN (34) or DD (36) for all n and all c, except n = 4, between c = 1.2 to 1.4, where SF (38) is more accurate than either of the other two.

CA: The LSN (40) and DD (42) are essentially identical for all n and all c. The SF was not run.

CB: The LSN (44) and DD (46) are essentially identical for all n and all c. The SF was not run.

AM-AD: The AM method is less accurate than AD in LSN (10, 19) and DD (13, 22) for all n = 2, 4, and 6, all c; for n = 8, c greater than 1.3, and for n = 12 and 16, c greater than about 1.6. For n = 12 and 16, c less than 1.6, AM is more accurate than AD.

AM-PN: The AM method is more accurate than PN for n = 4, all c, in LSN (10, 28) and DD (13, 30). For all n greater than 4, all c, PN is more accurate than AM.

AM-DPN: The DPN method is more accurate than AM in LSN (10, 34) and DD (13, 36) for all n, all c. This is not a surprising result.

AD-PN: The AD method is more accurate than PN in LSN (19, 28) and DD (22, 30) for n = 2, 4, 6, all c; and, for n = 8 above c = 1.4. For n = 12 and 16 and all c less than about 1.9, the PN is more accurate than AD.

AD-DPN: The DPN method is more accurate than AD in LSN (19, 34) and DD (22, 36) for all n, all c. This is not a surprising result.

PN-DPN: The DPN method is more accurate than PN in LSN (28, 34) and DD (30, 36) for all n, all c. This is not a surprising result.

AM-CA; The AM method is less accurate than CA in LSN (10, 40) and DD (13, 42) for all n, all c.

AD-CA: The AD method is more accurate than CA in LSN (18, 40) and DD (22, 42) for n = 4, all c; n = 6, greater than 1.5; and, n = 8, c greater than 1.4. Otherwise, CA is more accurate than AD.

PN-CA: The CA method is more accurate than PN in LSN (28, 40) and DD (30, 42) for n = 4, 6, 8, and c; for n = 12 and 16, c less than 1.8 For n = 12 and 16, c greater than 1.8, PN is more accurate than CA.

CA-CB: The CA method is more accurate than CB in LSN (40, 44) and DD (42, 46) for all n, all c.

0 CYLINDERS

AM: The LSN (11) is more accurate that DD (14) for all n, all c.The SF (17) is less accurate than either LSN (11) or DD (14) for all n, all c.

AD: The LSN (20) is more accurate than DD (23) for n = 2, 4, and 6, all c. For n greater than 6, the LSN (20) and DD (23) are essentially identical for all n, all c.

AM-AD: The AM is more accurate than AD for LSN (11, 20) and DD (14, 23) for all n, all c, except for n = 2. For n = 2 and c greater than 1.5, AD is more accurate than AM for both LSN (11, 20) and DD (14, 23).

0 SPHERES

AM: The LSN (12) is more accurate than DD (15) for n = 4, 6, 8, all c, and essentially identical for n = 12 and 16. For n = 2, DD (15) is more accurate than LSN (12), as is also the situation for n = 4 at c less than 1.05. The SF (18) is less accurate than n = 4 LSN (12) for all n, all c, except at n = 4, c = 1.2.

AD: The LSN (21) is more accurate than DD (24) for n = 4, 6, 8, all c and essentially identical for n = 12 and 16. For n = 2, DD (24) is more accurate than LSN (21), as is also the situation for n = 4 at c less than 1.05. The SF (27) is less accurate than n = 4 LSN (31) or DD (24) [except c = 1.2] for all n, all c.

PN: The LSN (29) is more accurate than DD (31) for n = 4, 6, 8, all c greater than 1.1, and essentially identical for n = 12 and 16, and c. The SF (33 is less accurate than n = 4 LSN (29) or DD (31) for all n, all c, except c = 1.2.

DPN: The LSN (35) is less accurate than DD (37) for all n, all c. The SF (39) is generally less accurate than n = 4 LSN (35) or DD (37) except between c = 1.1 to 1.4.

CA: The LSN (41) is more accurate than DD (43) for n = 4, all c; LSN (41) and DD (43) are essentially equal for n greater than 6, all c.

CB: The LSN (45) is more accurate than DD (47) for n = 4, all c; LSN (45) and DD (47) are essentially identical for n = 6, 8, and 12 for c greater than 1.2, below which LSN (45) is more accurate. LSN (45) is less accurate than DD (47) for n = 16, all c.

AM-AD: The AM is more accurate than AD for LSN (12, 21) and DD (15, 24) for all n, all c. Note that this is opposite the behavior in SLABS.

AM-PN: The AM is less accurate than PN for LSN (12, 29) and DD (15, 31) for all n, all c. This is the same result as for SLABS, except for n = 4.

AM-DPN: The AM is less accurate than DPN for LSN (12, 35) and DD (15, 37) for all n, all c.

AD-PN: The AD is less accurate than DPN for LSN (21, 29) and DD (24, 31) for all n, all c.

AD-DPN: The AD is generally less accurate than DPN for LSN (21, 35) and DD (24, 37) for all n, all c.

PN-DPN: The PN is less accurate than DPN for LSN (29, 35) and DD (31, 37) for all n, all c, except for c greater than 1.1 and n less than 8 and c less than 1.1.

Spheres (cont.)

AM-CA: The AM is less accurate than CA for LSN (12, 41) and DD (15, 43) for all n, all c.

AD-CA: The AD is less accurate than CA for LSN (21, 41) and DD (24, 43) for all n, all c, except for LSN (21, 41) with n less than 8 and c less than 1.1.

PN-CA: The PN is more accurate than CA for LSN (29, 41) and DD (31, 43) for all n, all c.

CA-CB: The CA is essentially identical to CB for LSN (41, 45) and DD (43, 47) for all n, all c; except for $n = 4$, where CA is more accurate than CB, and for $n = 16$, where CA is less accurate than CB.

VI.A.2 Summary of One-Group, One-Medium Problems

For slabs, the ordering of the direction sets in terms of decreasing accuracy appears to be DPN, (PN, CA, AD), AM, CB, where the (PN, CA, AD) sequence depends on which solution methods and value of c are being used. LSN and DD are generally equivalent in their predictions, except LSN does not generate negative fluxes whereas DD does, and it therefore requires a fix-up procedure.

For cylinders the LSN-AM method and direction sets should be used for best accuracy.

For spheres, the ordering of the directions sets in terms of decreasing accuracy seems to be DPN, PN, CA, AM, AD, and CB. The LSN method seems to give better results for n less than 12 than does the DD method if either PN, AM, or AD directions are used. If the DPN or CA directions are used, then the DD method is as accurate or more accurate than LSN. However, the user should be aware of the difficulties of negative flux generation in DD, which LSN automatically prevents while still giving excellent neutron balance.

In multidimensional problems, the DPN and PN directions are not available, so the user is limited to the sets AM, AD, CA, and CB. If n larger than 16 is to be run, and if CA and CB are not available, the direction sets by AM and AD are readily generated. It is simple to alter the selection procedure for determining μ_1^2; for example, one could arrange, using the subroutine supplied in Appendix A, to have a different direction set in each material region dependent on the region properties, provided the transport code could run with that assumption.

Finally, if one is interested in methods and directions uniformly giving an error less than 0.1% in critical dimensions for all c values, then consider:

<u>SLABS</u>

DPN-LSN (34) n greater than 8

DPN-DD (36) n greater than 8

SPHERES

PN-LSN (29)	n = 16, max error 0.15% at c = 2.0
PN-DD (31)	n = 16 max error 0.15% at c = 2.0
DPN-LSN (35)	n = 16
DPN-DD (37)	n = 16
CB-LSN (45)	n = 16
CB-DD (47)	n = 16

For methods and directions with 1% accuracy in the critical dimensions, examination of the results indicates a large number of potential candidates. However, for improvements at the 1% accuracy level in multidimensional calculations, without running very high order in calculations, significantly improved new methods will need to be developed.

VI.B. Lady Godiva Benchmark Problems

As an additional evaluation of the generated direction set and methods available in Discrete Ordinates Code, DSN,[11] modified to run with the Diamond Difference DD, pure Step Function on approximation SF, and the LSN[17] methods, we calculated the Lady Godiva Benchmark Problem.[14] This is a small spherical critical expermiment[15] of 8.71-cm radius composed of a homogeneous mixture of ^{235}U, with atomic densities of 0.045447 x 10^{24} and 0.00256 x 10^{24} atoms cm^3, respectively. The multigroup approximation using the Hansen-Roach[16] six-group cross sections and fission spectrum was used. The outside angular flux boundary condition, the Mark boundary condition (no incoming neutrons) was assumed. The relative changes in the pointwise scalar flux over the entire system and in the computed eigenvalue, k_{eff}, were required to be less than 10^{-6} for all calculations. With these inputs, we calculated the multiplication factor as a function of angular quadrature n and the number of space intervals I.

Previous results analyzing this benchmark problem solution were obtained and reported by Lathrop, Greenspan, Engle, and Whitesides in the

Benchmark book.[14] These results are outlined and summarized in Tables 48 and 49 for comparison purposes.

In Table 50 we have given the DSN results for LSN and Diamond Difference DD solutions for direction set determined by the Area Method (AM), Asymptotic Directions (AD), P_{n-1}(PN), $DP_{(n-2)/2}$ (DPN), and two EQ_n directions sets suggested by Carlson,[8] (CA) and (CB).

In Table 51 we compare the extrapolated k_{eff} [$I=\infty$, $n=\infty$] obtained by the various benchmark solutions with those found from Table 50. The interpolations in Table 51 were performed using $k = a + bM^{-2} + cM^{-4}$, where $M = In$ for sequences (b), (c), (d), and (e), and $M = I$ for the sequence (f), the integral transport solution. We note that even in the worst case of the comparison, [(e),(f) - (e)], there is still agreement of 0.001 in $\delta k/k$. This comparison indicates that the DPN and PN directions are more accurate than the others, which would be ranked CA, AM, CB, and AD in order of decreasing accuracy with respect to predicting the extrapolated critical k_{eff} for $I = \infty$, $n = \infty$. Comparison with the integral transport result [1 - A1 - 4, (f)] of $k_{eff} = 0.99597$ also indicates:

(1) The AM directions represent an improvement over the AD directions.

(2) The PN and DPN directions are most accurate in the limiting case.

(3) The CA is an improvement over the AM directions and almost as good as PN.

(4) The LSN and DD methods have comparable accuracy for a specified direction set.

The extrapolated k_{eff} comparison with benchmark solutions is useful in the limit. In standard production runs, however, the large values of I are run infrequently. It is, therefore, useful to compare the accuracy of the methods and directions sets in the finite parameter range frequently run by the computer codes. For this purpose, we form $\delta k/k$ from

TABLE 48

SUMMARY OF LADY GODIVA BENCHMARK SOLUTIONS

Benchmark Number	Author	Year	Code	Computer	Organization
1-A1-1	K.D. Lathrop	1966	DTF-IV	IBM-7030	Gulf General Atomic
1-A1-2	H. Greenspan	1966	SNARG1D	CDC-3600	Argonne Nat'l Laboratory
1-A1-3	W.W. Engle	1966	ANISN	IBM-7090 IBM-360/75	Union Carbide
1-A1-4	K.D. Lathrop	1967	MGSPIN	IBM-7030	Los Alamos Scientific Lab.
1-A1-5	G.E. Whitesides	1966	KENO	IBM-360/75 IBM-360/50	Union Carbide

TABLE 49

SUMMARY OF LADY GODIVA BENCHMARK PROBLEMS

1-A1-1 K.D. Lathrop, DTF-IV

I/n	4	8	16	32	64
10	1.00506	.998564	.996722	.996212	.996113
20	1.00514	.998564	.996675	.996179	.996079
40	1.00514	.998554	.996679	.996161	.996060
80	1.00514	.998550	.996674	.996155	.996055
160	1.00515	.998549	.996673	.996154	.996051

1-A1-2 H. Greenspan, SNARG 1-D

I/n	4	8	16	24
10	1.00615	.999034	.997066	.996595
20	1.00622	.999031	.997038	.996562
40	1.00623	.999019	.997020	.996543
80	1.00623	.999015	.997016	.996538
160	1.00623	.999015	.997015	-----

1-A1-3 W.W. Engle ANISN P_{n-1}

I	n	k_{eff}
10	4	1.00505
20	8	.998534
40	16	.996666
80	32	.996133
160	64	.995968

1-A1-4 K.D. Lathrop, Integral Solution

I	k_{eff}
20	.9955227
40	.9958613
80	.9959467
160	.9959661
320	.9959702

1-A1-5 G.E. Whitesides, KENO Monte Carlo

#Histories	k_{eff}
3,290,000	.99525±.00072
467,100	.99458±.00193
187,875	.99112±.00286
32,565	.99724±.00691

TABLE 50

LADY GODIVA k_{eff} FOR I SPACE CELLS ORDER S_n

I/n	4	8	16	32	64
(LSN,AM)					
10	1.00275	.999463	.999206	.997488	.996982
20	1.00412	.999575	.998750	.996973	.996579
40	1.00484	.999679	.997667	.996798	.996411
80	1.00524	.999779	.997661	.996760	.996356
160	1.00526	.999850	.997675	.996753	.996343
(DD,AM)					
10	1.00562	.999960	.997757	.996821	.996403
20	1.00569	.999959	.997733	.996789	.996369
40	1.00570	.999949	.997717	.996771	.996350
80	1.00570	.999946	.997712	.996766	.996345
160	1.00570	.999945	.997711	.996764	.996343
(LSN,AD)					
10	1.00338	1.00066	.999084	.997981	.997228
20	1.00473	1.00057	.998494	.997410	.996806
40	1.00540	1.00064	.998348	.997199	.996623
80	1.00578	1.00071	.998328	.997142	.996558
160	1.00600	1.00077	.998344	.997132	.996540
(DD,AD)					
10	1.00615	1.00087	.998408	.997193	.996597
20	1.00612	1.00087	.998385	.997162	.996563
40	1.00623	1.00086	.998370	.997145	.996545
80	1.00623	1.00085	.998365	.997140	.996539
160	1.00623	1.00085	.998364	.997138	.996538
(LSN,PN)					
10	1.00239	.999271	.997869	.996988	.996509
20	1.00367	.998631	.997079	.996481	.996220
40	1.00433	.998507	.996786	.996270	.996095
80	1.00471	.998491	.996697	.996192	.996046
160	1.00492	.998507	.996675	.996165	.996028
(DD,PN)					
10	1.00506	.998564	.996722	.996213	.996079
20	1.00514	.998564	.996696	.996180	.996044
40	1.00514	.998554	.996680	.996161	.996025
80	1.00515	.998551	.996675	.996156	.996020
160	1.00515	.998550	.996674	.996155	.996019
(LSN,DPN)					
10	1.00498	.999655	.997705	.996811	.996406
20	1.00468	.998460	.996935	.996375	.996161
40	1.00471	.998027	.996636	.996201	.996061
80	1.00476	.997885	.996529	.996138	.996025
160	1.00480	.997844	.996492	.996116	.996012
(DD,DPN)					
10	1.00480	.997834	.996516	.996161	.996066
20	1.00489	.997843	.996495	.996130	.996032
40	1.00490	.997836	.996480	.996112	.996013
80	1.00490	.997833	.996476	.996107	.996008
160	1.00490	.997832	.996474	.996105	.996006

TABLE 50 (continued)

(LSN,CA)					
10	1.00275	.999575	.998045	----	----
20	1.00410	.999122	.997261	----	----
40	1.00477	.999069	.996986	----	----
80	1.00516	.999086	.996911	----	----
160	1.00537	.999117	.996894	----	----
(DD,CA)					
10	1.00553	.999189	.996947	----	----
20	1.00560	.999189	.996921	----	----
40	1.00561	.999179	.996905	----	----
80	1.00561	.999176	.996900	----	----
160	1.00561	.999175	.996899	----	----
(LSN,CB)					
10	1.00338	.999509	.997411	----	----
20	1.00473	.999059	.996630	----	----
40	1.00540	.999007	.996356	----	----
80	1.00578	.999024	.996282	----	----
160	1.00600	.999055	.996265	----	----
(DD,CB)					
10	1.00615	.999128	.996316	----	----
20	1.00622	.999127	.996292	----	----
40	1.00623	.999117	.996276	----	----
80	1.00623	.999114	.996271	----	----
160	1.00623	.999113	.996270	----	----

TABLE 51

SUMMARY OF EXTRAPOLATED LADY GODIVA BENCHMARK

K_{eff}

Method	(a)	(b)	(c)	(d)	(e)	f-b x10[5]	f-c x10[5]	f-d x10[5]	f-e x10[5]
1-A1-1	.99604	.99602	.99598	.99603	.99603	-5	-1	-6	-6
1-A1-2	.99630	----	----	.99633	.99633	---	---	-36	-36
1-A1-3	.99597	.99591	----	----	.99602	+6	---	---	-5
1-A1-4[f]	.99597	.99597	.99597	.99597	.99597	-0-	-0-	-0-	-0-
1-A1-5	.99525±.00072								
LSN,AM	----	.99619	.99641	.99690	.99693	-22	-44	-93	-96
DD,AM	----	.99619	.99641	.99690	.99690	-22	-44	-93	-93
LSN,AD	----	.99631	.99668	.99745	.99747	-34	-71	-148	-150
DD,AD	----	.99631	.99668	.99743	.99743	-34	-71	-146	-146
LSN,PN	----	.99597	.99598	.99604	.99609	-0-	-1	-7	-12
DD,PN	----	.99597	.99598	.99603	.99603	-0-	-1	-6	-6
LSN,DPN	----	.99597	.99597	.99606	.99601	-0-	-0-	-9	-4
DD,DPN	----	.99597	.99598	.99606	.99606	-0-	-1	-9	-9
LSN,CA	----	----	----	.99609	.99616	---	---	-12	-19
DD,CA	----	----	----	.99608	.99608	---	---	-11	-11
LSN,CB	----	----	----	.99524	.99531	---	---	+73	+66
DD,CB	----	----	----	.99523	.99523	---	---	+74	+74

a. Benchmark Problem Book Results, Ref. (14).
b. (I,n) = (40,16), (80,32), (160,64)
c. (I,n) = (40, 8), (80,16), (160,32)
d. (I,n) = (40, 4), (80, 8), (160,16)
e. (I,n) = (10, 4), (20, 8), (40,16)
f. (I) = (80, 160, 320) = (40, 80, 160) in 1-A1-4.

the integral transport result for fixed spatial intervals and various n. Thus, in Table 52 we compare

$$\delta k/k = 1 - k(I,n, \text{Table } 50)/k(I,1\text{-A1-4}) \quad . \qquad (41)$$

This will specifically address the relative accuracy of the Discrete Ordinates method at a specified spatial representation relative to the integral transport solution for a specified n. In Table 52 we observe that

(1) 1-A1-1 and DD, PN are almost identical.

(2) 1-A1-2 (PN) is less accurate than DD, PN for all n.

(3) 1-A1-3 (PN), for the sequence given, is more accurate than DD, PN (LSN, DD, LSN, DPN) for the same sequence. It is less accurate than DD, DPN except at I = 160, N = 64.

(4) LSN is more accurate than DD for n = 4, 8 in AM, AD, CA, and CB.

(5) LSN is more accurate than DD for n = 4 in PN and DPN.

(6) For n greater than 8, LSN is less or equally accurate than DD in AM, AD, PN, DPN, CA, and CB.

(7) LSN, DPN and DD, DPN are more accurate than 1-A1-1, LSN, PN and DD, PN for all n.

(8) DD, DPN is more accurate than LSN, DPN for all I, all n.

(9) LSN, CA (DD, CB) is more accurate than LSN, CB (DD, CB) for n = 8 and 16. For n = 4, LSN , CA is more accurate than LSN, CB, but DD, CA is less accurate than DD, CB.

TABLE 52

COMPARISON OF S_n K_{eff} CALCULATIONS VERSUS INTEGRAL SOLUTIONS FOR THE SAME NUMBER OF SPATIAL INTERVALS, I

$\delta k_{eff}/k_{eff} \times 10^3$

I/n	4	8	16	32	64
1-A1-1					
20	9.681	3.055	1.157	.6593	.5588
40	9.317	2.704	.8211	.3009	.1995
80	9.231	2.614	.7303	.2091	.1087
160	9.221	2.593	.7098	.1887	.0852
1-A1-2					
20	10.745	3.524	1.522	---	---
40	10.412	3.171	1.164	---	---
80	10.325	3.081	1.074	---	---
160	10.305	3.061	1.053	---	---
1-A1-3					
20	---	3.025	---	---	---
40	---	---	.808	---	---
80	---	---	---	.1887	---
160	---	---	---	---	.0019
LSN,AM					
20	8.636	4.071	3.243	1.457	1.061
40	9.016	3.834	1.813	.9406	.5520
80	9.331	3.848	1.721	.8166	.4110
160	9.332	3.900	1.716	.7901	.3784
DD,AM					
20	10.213	4.456	2.220	1.272	.8501
40	9.880	4.105	1.863	.9135	.4907
80	9.793	4.016	1.772	.8226	.3999
160	9.773	3.995	1.752	.8011	.3784
LSN,AD					
20	9.249	5.070	2.985	1.986	1.289
40	9.578	4.799	2.497	1.343	.7649
80	9.873	5.019	2.519	1.200	.6138
160	10.075	4.823	2.388	1.171	.5762
DD,AD					
20	10.645	5.371	2.875	1.647	1.045
40	10.412	5.019	2.519	1.289	.6865
80	10.325	4.923	2.428	1.198	.5947
160	10.305	4.904	2.408	1.177	.5742
LSN,PN					
20	8.184	3.122	1.563	.9626	.7004
40	8.504	2.657	.9285	.4104	.2347
80	8.799	2.555	.7534	.2463	.0997
160	8.990	2.551	.7118	.1997	.0622

TABLE 52. (Continued)

DD,PN					
20	9.661	3.055	1.179	.6603	.5236
40	9.317	2.704	.8221	.3009	.1644
80	9.241	2.615	.7313	.2102	.0736
160	9.221	2.594	.7108	.1897	.0531
LSN,DPN					
20	9.198	2.951	1.419	.8561	.6412
40	8.885	2.175	.7779	.3411	.2005
80	8.849	1.946	.5847	.1921	.0786
160	8.870	1.886	.5280	.1505	.0461
DD,DPN					
20	9.409	2.331	.9767	.6100	.5116
40	9.076	1.983	.6213	.2517	.1523
80	8.990	1.894	.5315	.1610	.0616
160	8.970	1.873	.5100	.1395	.0401
LSN,CA					
20	8.616	3.615	1.746	---	---
40	8.946	3.221	1.129	---	---
80	9.251	3.152	.9682	---	---
160	9.442	3.164	.9317	---	---
DD,CA					
20	10.123	3.683	1.405	---	---
40	9.789	3.331	1.048	---	---
80	9.703	3.242	.9572	---	---
160	9.683	3.222	.9367	---	---
LSN,CB					
20	9.249	3.552	1.112	---	---
40	9.578	3.159	.4968	---	---
80	9.873	3.090	.3367	---	---
160	10.075	3.101	.3001	---	---
DD,CB					
20	10.745	3.621	.7728	---	---
40	10.412	3.269	.4164	---	---
80	10.325	3.180	.3256	---	---
160	10.305	3.160	.3051	---	---

(10) All the direction sets yield $\delta k/k$ of the order (all I)

(a) $8 - 10 \times 10^{-3}$ for $n = 4$,

(b) $2 - 4 \times 10^{-3}$ for $n = 8$,

(c) $0.3 - 3 \times 10^{-3}$ for $n = 16$,

(d) $0.2 - 2 \times 10^{-3}$ for $n = 32$, and

(e) $0.02 - 1.3 \times 10^{-3}$ for $n = 64$.

The corresponding values for DPN are 9 - 9.4×10^{-3}, 1.8 - 2.3×10^{-3}, 0.51 - 98×10^{-3}, 0.14 - 0.61×10^{-3}, and 0.04 - 0.51×10^{-3}, respectively.

(11) The differences in LSN and DD for the same direction set are due to the different solution algorithms used. LSN always gives nonnegative extrapolated fluxes and exact neutron conservation, whereas DD predicts some (usually few in these problems) negative extapolated fluxes that must be "fixedup" or "set to zero." Inconsistent and/or artificial modifications of the original difference operator can produce nonconservation of neutrons in the system. Additionally, such procedures destroy reciprocity, orthogonality of the flux and adjoint solutions, and preclude the equality of the flux and adjoint eigenvalues. LSN maintains the physical properties of neutron conservations, flux positivity, and equality of flux and adjoint eigenvalues, but at the expense of the eigenvalue accuracy for a specified approximation comparison. That results because LSN uses a variable weighting scheme, different in each phase dimension, which always lies between DD (2^{nd} order) and SF (1^{st} order), and guarantees positivity of the extrapolated angular fluxes.

Examination of the computing times of the benchmark problems indidicates that they represent a significant effort of the computers then available. In Table 53, we indicate a comparison of the computing speeds on the Lady Godiva $[I = 40, n = 16]$ problem.

The single case given in Table 50 for Lady Godiva represented about $1/60^{th}$ of the total computing effort necessary to generate the values I = 10, 20, 40, 80, 160 versus n = 4, 8, 16, 32, 64.

TABLE 53
RELATIVE COMPUTATION SPEED OF THE
LADY GODIVA BENCHMARK CALCULATIONS

Machine	Total Time	Inverse Relative Speed
IBM-7090	5 min	20.8
IBM-7030	3 min	12.5
CDC-3600	2.15 min	9.0
IBM-360/75	1.32 min	5.5
CDC-7600	14.4 sec	1.0

VII. CONCLUSIONS

We have formulated and compared a new quadrature technique on the unit sphere with applications to the Discrete Ordinate S_n transport theory methods. A computer program, listed in Appendix A, provides for the rapid generation of weights and directions at run time for arbitrary order n, as needed by the one- and two-dimensional transport codes. Positive point weights are generated for arbitrary order n less than 10 000. Typical computer times are less than a microsecond per diagram on the octant on a CDC-7600.

Typically, a full case of the one-group problems in Tables 10-47 requires about four minutes on the CDC-7600.

In general, the AM direction in cylinders and spheres yields more accurate results than the ADs for the LSN and DD solution methods. The AM direction sets are directly usable by the multidimensional discrete ordinate transport codes. The P_{n-1}, $DP_{n-2)/2}$, and EQ_n directions exhibit greater accuracy than AM but do not (except for EQ_n) extend to multidimensional systems.

In the AM and AD method direction sets studies presented here, we have restricted our selection of μ_1 to depend only on n. These results are, therefore, best only for c - 1 near zero. This was done primarily to compare with results obtained previously in a uniform manner.

It is well known that μ_1 should depend on c - 1, even in S_2 approximations,[21-23] if more accurate solutions are to be obtained. For values of c - 1 not near zero, even as n increases, the errors of the critical radius in slab geometry increase. It is now possible to investigate alternative direction set representation automatically, using the program in Appendix A for higher order n values. If different direction sets are allowed in each material region of an S_n calculation, then the available program packages will require modification; however, some versions already exist with this feature available. If the system average c is used, then the program packages in current usage would be easily modified by specifying a single direction set at run time.

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APPENDIX A

COMPUTER PROGRAM LISTING

AREA WEIGHTS AND DIRECTIONS

LOS ALAMOS IDENTIFICATION NO. LP-1550

```
      PROGRAM AMWD (OUT)
      PARAMETER (NMOST=80),(NHALF=NMOST/2),(KMOST=3*NHALF),(MOSTQ=2*NMOS
     1T+1)
C
C     AREA METHOD WEIGHTS AND DIRECTIONS CODE.
C
C     THIS PROGRAM WAS WRITTEN IN JUNE 1975 BY
C               L. M. CARRUTHERS
C               C. E. LEE
C     AT LOS ALAMOS SCIENTIFIC LABORATORY.
C
      DIMENSION W(NMOST), UB(NMOST), Q(MOSTQ)
      DIMENSION NSET(16)
      DATA NSET/2,4,6,8,10,12,14,16,20,24,28,32,40,48,56,64/
      COMMON IGE
      DIMENSION WO(250)
      DO 20 INS=1,16
      N=NSET(INS)
      PRINT 30
      IGE=1
      PRINT 40, N,IGE
      PRINT 50
      CALL CSNC (N,W,UB)
      NH=N/2
      DO 10 I=1,NH
      INDEX=NH+1+I
   10 WO(I)=W(INDEX)*W(INDEX)
      CALL CSNW (N,WO,UB(NH+2),Q)
      IF (INS.GT.8) GO TO 20
      IGE=2
      PRINT 40, N,IGE
      PRINT 60
      CALL CSNC (N,W,UB)
   20 CONTINUE
      CALL EXIT
C
   30 FORMAT (1H1)
   40 FORMAT (4H N =,I2,3X,5HIGE =,I1)
   50 FORMAT (1H+,18X,17HSLABS AND SPHERES/)
   60 FORMAT (1H+,18X,9HCYLINDERS/)
      END
      SUBROUTINE CSNC(N,W,UB)
      PARAMETER (NMOST=80),(NHALF=NMOST/2),(KMOST=3*NHALF),(MOSTQ=2*NMOS
     1T+1)
      DIMENSION W(1), UB(1)
      DIMENSION WW(NHALF,NHALF)
      DOUBLE PRECISION U
      COMMON /UU/ U(KMOST)
      COMMON IGE,K3
      UONE=SQRT(2./(3.*N))
      XN=N
C     ASYMPTOTIC DIRECTIONS OF LA-2595
      UN=1.0/SQRT(3.0*XN-3.0)
C     AREA METHOD DIRECTIONS
      UN=0.69/SQRT(XN)-.001
      UN2=UN*UN
      K1=3*N/2-2
      NH=N/2
      K3=N+1
      NC=NH*(NH+2)
```

```
C       ZERO THE ARRAY FOR WEIGHTS
        DO 10 I=1,NH
        W(I)=0.0
   10   CONTINUE
C       TRIPLE LEVEL SPACING
        DEL1=0.0
        IF (N.NE.2) DEL1=2./(N-2.)*(1.-3.*UONE*UONE)
        DEL=DEL1/3.
C       POINT COSINES
        DO 20 I=1,K1
        U(I)=SQRT(UONE*UONE+(I-1)*DEL)
   20   CONTINUE
C       THE (K1+1)ST ELEMENT IS USED AS THE ZEROTH ELEMENT
        U(K1+1)=0.0
        U(K1+2)=1.0
C       LEVEL COSINES
        IF (NH.EQ.1) GO TO 240
        UB(1)=UN
        DO 30 I=2,NH
        UB(I)=SQRT((I-1)*(1.-3.*UN2)/(NH-1.)+UN2)
   30   CONTINUE
        NH1=NH-1
        IF (N-6) 220,180,40
   40   NR=MOD(NH1,3)+1
        GO TO (50,60,70), NR
   50   I=NH
        J=NH
        GO TO 80
   60   I=NH1
        J=NH1
        GO TO 80
   70   I=NH-2
        J=NH+1
   80   P=HEXA(N,I,J)
        K=3*NH-I-J
        LI1=I/3
        LI=LI1+1
        IF (I.EQ.J) GO TO 120
        LJ1=J/3
        LJ=LJ1+1
        IF (J.EQ.K) GO TO 100
        LK1=K/3
        LK=LK1+1
        IF (IGE.EQ.2) GO TO 90
        P2=P+P
        W(LI)=W(LI)+P2
        W(LJ)=W(LJ)+P2
        W(LK)=W(LK)+P2
        GO TO 160
   90   WW(LI1,LJ)=P
        WW(LI1,LK)=P
        WW(LJ1,LI)=P
        WW(LJ1,LK)=P
        WW(LK1,LI)=P
        WW(LK1,LJ)=P
        GO TO 160
  100   IF (IGE.EQ.2) GO TO 110
        W(LI)=W(LI)+P
        W(LJ)=W(LJ)+P+P
        GO TO 160
  110   WW(LI1,LJ)=P
        WW(LJ1,LI)=P
        WW(LJ1,LJ)=P
```

```
      GO TO 160
  120 IF (J.NE.K) GO TO 140
      IF (IGE.EQ.2) GO TO 130
      W(LI)=W(LI)+P
      GO TO 160
  130 WW(LI1,LI)=P
      GO TO 160
  140 LK1=K/3
      LK=LK1+1
      IF (IGE.EQ.2) GO TO 150
      W(LI)=W(LI)+P+P
      W(LK)=W(LK)+P
      GO TO 160
  150 WW(LI1,LI)=P
      WW(LI1,LK)=P
      WW(LK1,LI)=P
  160 IF (I.EQ.J) GO TO 170
      J=J-3
      GO TO 80
  170 IF (I.EQ.4) GO TO 180
      I=I-3
      J=(3*NH-I)/2
      J=J-MOD(J-1,3)
      GO TO 80
  180 J=1+3*(NH1/2)
  190 P=PENTA(N,J)
      K=3*NH-1-J
      M=2
      IF (J.EQ.K) M=1
      LJ1=J/3
      LJ=LJ1+1
      LK1=K/3
      LK=LK1+1
      IF (IGE.EQ.2) GO TO 200
      PM=M*P
      W(1)=W(1)+PM
      W(LJ)=W(LJ)+PM
      W(LK)=W(LK)+PM
      GO TO 210
  200 W(LJ)=P
      WW(LJ1,1)=P
      WW(LJ1,LK)=P
      IF (LJ.EQ.LK) GO TO 210
      W(LK)=P
      WW(LK1,1)=P
      WW(LK1,LJ)=P
  210 IF (J.EQ.4) GO TO 220
      J=J-3
      GO TO 190
  220 P=QUAD(N)
      W(NH)=P
      IF (IGE.EQ.2) GO TO 230
      W(1)=W(1)+P+P
      GO TO 250
  230 W(1)=P
      WW(NH-1,1)=P
      GO TO 250
  240 CONTINUE
      UB(1)=1.0/SQRT(3.0)
      W(1)=1.0
  250 CONTINUE
      DO 260 I=1,NH
      IP=1+NH+I
```

```
      W(IP)=W(I)*0.5
      UB(IP)=UB(I)
  260 CONTINUE
      DO 270 I=1,NH
      IP=NH+1+I
      IM=NH+2-I
      W(IM)=W(IP)
      UB(IM)=-UB(IP)
  270 CONTINUE
      W(1)=0.0
      UB(1)=-1.0
      IF (IGE.NE.2) GO TO 300
      UB(1)=-SQRT(1.0-UB(NH+2)**2)
      IF (N.EQ.2) GO TO 300
      NH1=NH-1
      DO 290 J=1,NH1
      K31=K3+1
      W(K31)=0.0
      UB(K31)=-SQRT(1.0-UB(J+NH+2)**2)
      NHI=NH-J
      DO 280 I=1,NHI
      IL=1+NH+I
      IP=K3+IL-J
      IM=K3+2+NHI-I
      W(IP)=WW(J,I)*0.5
      UB(IP)=UB(IL)
      W(IM)=W(IP)
      UB(IM)=-UB(IP)
  280 CONTINUE
      K3=K3+NHI+NHI+1
  290 CONTINUE
  300 CONTINUE
      WSUM=0.0
      DO 310 I=1,K3
      WSUM=WSUM+W(I)
      IF (W(I).LT.0.0) PRINT 340, N,I,W(I)
  310 CONTINUE
      IF (WSUM.EQ.1.0) GO TO 330
      WNORM=1.0/WSUM
      DO 320 I=1,K3
  320 W(I)=W(I)*WNORM
  330 CONTINUE
      PRINT 350, (I,W(I),I,UB(I),I=1,K3)
      IF (IGE.NE.2) RETURN
      IF (K3.NE.NC) CALL EXIT
      RETURN
C
  340 FORMAT (8H FOR N =,I7,3H W(,I7,3H) =,E15.5)
  350 FORMAT (3H W(,I3,2H)=,F20.15,5X,3HUB(,I3,2H)=,F20.15)
      END
      SUBROUTINE CSNW(N,W,UB,Q)
      DIMENSION W(1), UB(1), Q(1)
      K2=N/2
      K3=2*N+1
C     ZERO MOMENT ARRAY
      DO 10 I=1,K3
      Q(I)=0.0
   10 CONTINUE
C     COMPUTE 0 TO 2N MOMENTS  W(J)*UB(J)**K
      SUMQ=0.0
      DO 30 I=1,K3
      SUMA=0.0
      DO 20 J=1,K2
```

```
      SUMA=SUMA+W(J)*(UB(J)**(I-1))
   20 CONTINUE
      Q(I)=(SUMA*I-1.0)*100.
      SUMQ=SUMQ+ABS(Q(I))
   30 CONTINUE
      PRINT 40, N,Q(1),(I,Q(I+1),Q(I+K2+1),Q(I+2*K2+1),Q(I+3*K2+1),I=1,K
     12)
      SUM=SUMQ/K3
      PRINT 50, SUM
      RETURN
C
   40 FORMAT (///8H MOMENTS//4H N =,I5,5X,5HQ(1)=,E11.4//4H   I,9X,6HQ(I
     1+1),5X,9HQ(I+K2+1),3X,11HQ(I+2*K2+1),3X,11HQ(I+3*K2+1)//(1X,I3,1X,
     24F14.4))
   50 FORMAT (1H0,46HAVERAGE ABSOLUTE PERCENTAGE ERROR OF MOMENTS =,4X,F
     110.4////)
      END
      FUNCTION QUAD (N)
      PARAMETER (NMOST=80),(NHALF=NMOST/2),(KMOST=3*NHALF),(MOSTQ=2*NMOS
     1T+1)
      DOUBLE PRECISION U,ANG,ALP,GCOS,S12,S13,S14,S23,S24,S34
      DOUBLE PRECISION E,B,C
      COMMON /UU/ U(KMOST)
      DIMENSION A(3)
      ANG(N1,N2,N3,N4,N5,N6)=(U(N1)*U(N2)+U(N3)*U(N4)+U(N5)*U(N6))
      ALP(E,B,C)=.6366197723564810*GCOS((C-E*B)/(DSQRT(1.0D0-E*E)*DSQRT(
     11.0D0-B*B)))
      N1=2
      N2=3
      N3=3*N/2-4
      N4=N3+1
      N5=N3+3
      N6=N3+4
      S12=ANG(N1,N2,N1,N5,N3,N4)
      S13=ANG(N2,N5,N5,N2,N4,N4)
      S14=ANG(N2,N5,N5,N5,N4,N6)
      S23=ANG(N1,N2,N1,N5,N3,N4)
      S24=ANG(N1,N5,N1,N5,N3,N6)
      S34=ANG(N2,N5,N5,N5,N4,N6)
      A(1)=ALP(S12,S23,S13)
      A(2)=ALP(S14,S12,S24)
      A(3)=ALP(S34,S14,S13)
      QUAD=A(1)+A(2)+A(2)+A(3)-4.
      DO 10 I=1,3
      IF (A(I).LT.0.0) PRINT 20, N,I,A(I)
   10 CONTINUE
      IF (QUAD.LT.0.0) PRINT 30, N,QUAD,A
      RETURN
C
   20 FORMAT (8H FOR N =,I7,3H A(,I7,3H) =,E15.5)
   30 FORMAT (3H N=,I5,6H QUAD=,F20.15/3F20.15)
      END
      FUNCTION PENTA (N,J)
      PARAMETER (NMOST=80),(NHALF=NMOST/2),(KMOST=3*NHALF),(MOSTQ=2*NMOS
     1T+1)
      DIMENSION A(5)
      DOUBLE PRECISION U,ANG,ALP,GCOS,S12,S13,S14,S15,S23,S24,S25,S34,S3
     15,S45
      DOUBLE PRECISION E,B,C
      COMMON /UU/ U(KMOST)
      ANG(N1,N2,N3,N4,N5,N6)=(U(N1)*U(N2)+U(N3)*U(N4)+U(N5)*U(N6))
      ALP(E,B,C)=.6366197723564810*GCOS((C-E*B)/(DSQRT(1.0D0-E*E)*DSQRT(
     11.0D0-B*B)))
```

```
      N1=2
      N2=3
      N3=J-2
      N4=J-1
      N5=J+1
      N6=J+2
      N7=3*N/2-J-3
      N8=N7+1
      N9=N7+3
      N10=N7+4
      N11=N7+J+2
      S12=ANG(N1,N2,N5,N4,N7,N8)
      S13=ANG(N2,N11,N4,N6,N8,NA)
      S14=ANG(N2,N11,N4,N4,N8,N10)
      S15=ANG(N1,N2,N3,N4,N9,N8)
      S23=ANG(N1,N11,N5,N6,N7,NA)
      S24=ANG(N1,N11,N5,N4,N7,N10)
      S25=ANG(N1,N1,N3,N5,N9,N7)
      S34=ANG(N4,N6,N10,NA,N11,N11)
      S35=ANG(N1,N11,N3,N6,N9,NA)
      S45=ANG(N1,N11,N3,N4,N9,N10)
      A(1)=ALP(S15,S12,S25)
      A(2)=ALP(S12,S23,S13)
      A(3)=ALP(S23,S34,S24)
      A(4)=ALP(S34,S45,S35)
      A(5)=ALP(S45,S15,S14)
      PENTA=A(1)+A(2)+A(3)+A(4)+A(5)-6.
      DO 10 I=1,5
      IF (A(I).LT.0.0) PRINT 20, N,I,A(I)
   10 CONTINUE
      IF (PENTA.LT.0.0) PRINT 30, J,N,PENTA,A
      RETURN
C
   20 FORMAT (AH FOR N =,I7,3H A(,I7,3H) =,E15.5)
   30 FORMAT (3H J=,I4,3H N=,I5,7H PENTA=,F20.15/3F20.15/2F20.15)
      END
      FUNCTION HEXA (N,I,J)
      PARAMETER (NMOST=80),(NHALF=NMOST/2),(KMOST=3*NHALF),(MOSTQ=2*NMOS
     1T+1)
      DOUBLE PRECISION U,ANG,ALP,GCOS,S12,S13,S15,S16,S23,S24,S26,S34,S3
     15,S45,S46,S56
      DOUBLE PRECISION E,B,C
      DIMENSION A(6)
      COMMON /UU/ U(KMOST)
      ANG(N1,N2,N3,N4,N5,N6)=(U(N1)*U(N2)+U(N3)*U(N4)+U(N5)*U(N6))
      ALP(E,B,C)=.6306197723564810*GCOS((C-E*B)/(DSWRT(1.0D0-E*E)*DSQRT(
     11.0D0-B*B)))
      N1=I-2
      N2=I-1
      N3=I+1
      N4=I+2
      N5=J-2
      N6=J-1
      N7=J+1
      N8=J+2
      N9=3*N/2-I-J-2
      N10=N9+1
      N11=N9+3
      N12=N9+4
      S12=ANG(N3,N4,N7,N6,N9,N10)
      S13=ANG(N2,N4,N8,N6,N10,N10)
      S15=ANG(N2,N4,N6,N6,N12,N10)
      S16=ANG(N3,N4,N5,N6,N11,N10)
```

```
      S23=ANG(N3,N2,N7,N8,N9,N10)                                        AMWD
      S24=ANG(N1,N3,N7,N7,N11,N9)                                        AMWD
      S26=ANG(N3,N3,N5,N7,N11,N9)                                        AMWD
      S34=ANG(N1,N2,N7,N8,N11,N10)                                       AMWD
      S35=ANG(N2,N2,N6,N8,N12,N10)                                       AMWD
      S45=ANG(N1,N2,N7,N6,N11,N12)                                       AMWD
      S46=ANG(N1,N3,N7,N5,N11,N11)                                       AMWD
      S56=ANG(N3,N2,N5,N6,N11,N12)                                       AMWD
      A(1)=ALP(S16,S12,S26)                                              AMWD
      A(2)=ALP(S12,S23,S13)                                              AMWD
      A(3)=ALP(S23,S34,S24)                                              AMWD
      A(4)=ALP(S34,S45,S35)                                              AMWD
      A(5)=ALP(S45,S56,S46)                                              AMWD
      A(6)=ALP(S56,S16,S15)                                              AMWD
      HEXA=A(1)+A(2)+A(3)+A(4)+A(5)+A(6)-8.                              AMWD
      DO 10 M=1,6                                                        AMWD
      IF (A(M).LT.0.0) PRINT 20, N,M,A(M)                                AMWD
   10 CONTINUE                                                           AMWD
      IF (HEXA.LT.0.0) PRINT 30, I,J,N,HEXA,A                            AMWD
      RETURN                                                             AMWD
C                                                                        AMWD
   20 FORMAT (8H FOR N =,I7,3H A(,I7,3H) =,E15.5)                        AMWD
   30 FORMAT (3H I=,I4,3H J=,I4,3H N=,I5,5H HEX=,F20.15/3F20.15/3F20.15) AMWD
      END                                                                AMWD
      DOUBLE PRECISION FUNCTIONGCOS(V)                                   AMWD
      DOUBLE PRECISION V,U,PI                                            AMWD
      DATA PI/3.141592653589793/                                         AMWD
      U=V                                                                AMWD
      IF (U.EQ.0.0) GO TO 10                                             AMWD
      GCOS=DATAN(DSQRT(1.0D0-U*U)/U)                                     AMWD
      IF (GCOS.LT.0.0) GCOS=GCOS+PI                                      AMWD
      RETURN                                                             AMWD
   10 GCOS=0.5D0*PI                                                      AMWD
      RETURN                                                             AMWD
      END                                                                AMWD
```


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